Active vibration control of a light and flexible stress ribbon footbridge using pneumatic muscles

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Abstract: This paper describes the development of an active vibration control system for a light and flexible stress ribbon footbridge. The 13 m span Carbon Fiber Reinforced Plastics (CFRP) stress ribbon bridge was built in the laboratory of the Department of Civil and Structural Engineering, Berlin Institute of Technology. Its lightness and flexibility result in high vibration sensitivity. To reduce pedestrian-induced vibrations, very light pneumatic muscle actuators are placed at handrail level introducing control forces. First, a reduced discretized analytical model is derived for the stress ribbon bridge. To verify the analytical prediction, experiments without feedback control are conducted. Based on this model, a velocity feedback control strategy is designed to actively control first mode vibrations. To handle the nonlinearities of the muscle actuator a subsidiary nonlinear force controller is implemented based on exact linearisation methods. The stability of the entire closed-loop system with actuator saturation is investigated by the Popov Criterion. Control performance is verified by experiments. It is demonstrated that handrail introduced forces can efficiently control the first mode response.

Keywords: active vibration control; light-weight structure; stress ribbon bridge; pedestrian induced vibration; pneumatic muscle actuator; velocity feedback control; exact linearization

1. INTRODUCTION

Stress ribbon bridges are among the most elegant and lightest bridges. Due to their static and dynamic characteristics, they have been mainly designed for pedestrian traffic rather than for road or rail traffic (Strasky (2005), Baus et al. (2008)). The suspension cable and the bridge deck are combined into one load-bearing element, which is anchored in the abutments. Usually the ribbons are made of steel cables or steel plates. To show the potential of high-strength Carbon Fiber Reinforced Plastics (CFRP), a 13 m span stress ribbon bridge with CFRP ribbons was built in the laboratory of the Department of Civil and Structural Engineering, TU Berlin by Schlaich et al. (2007). This composite material allows considerably smaller cross sections. The bridge’s tensile force under dead and live load is carried by only six ribbons with a cross section of ≈1.1 x 50 mm each. The combination of low extensional stiffness using CFRP for the ribbons and a light-weight bridge deck leads to considerable dynamic responses caused by pedestrian loads. In order to reduce the vibration amplitudes a multi-modal active vibration control (AVC) system was developed (Bleicher (2011)). In particular it is necessary for extremely light structures, where the system properties like mass and stiffness become time-variant by changing pedestrian traffic. In this paper, the results of the AVC system to reduce first mode responses are presented. Therefore, control forces are introduced on handrail level by two very light pneumatic muscle actuators (PMA) placed in midspan (Fig. 1, Fig. 2).

Fig. 1. Concept of active vibration control for the first mode.

Fig. 2. Equipment to control the force of the PMA
To design a model-based velocity feedback controller an analytical model is derived for the bridge in Section 2. The model is also used to estimate modal states with a Kalman filter, which is described in Section 3. An analytical nonlinear model of the actuator system is developed in Section 4. Based on this model an I/O linearization controller design is applied for the PMA in Section 5. As a result the actuator dynamics becomes a first order system. The velocity feedback controller is designed in Section 6. In Section 7, experimental results of the cascaded active vibration control are shown.

2. ANALYTICAL MODEL OF THE STRESS RIBBON BRIDGE

From the distributed system of the stress ribbon bridge, an analytical plane rigid body model for multi-variable control system design is developed (cf. Fig. 3 and Bleicher (2011)). In order to get good agreement with experimental data for the modes to be controlled a plane model with seven degrees of freedom \( q_i \) is developed using Euler-Lagrange equations. The parameters of the 8-plate model are listed in Table 1.

Table 1. Parameters of the 8-plate model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total bridge length</td>
<td>13.05 m</td>
</tr>
<tr>
<td>Plate length</td>
<td>1.63 m</td>
</tr>
<tr>
<td>Plate thickness</td>
<td>0.10 m</td>
</tr>
<tr>
<td>Total mass</td>
<td>4336 kg</td>
</tr>
<tr>
<td>Plate mass</td>
<td>542 kg</td>
</tr>
<tr>
<td>Mass moment of inertia</td>
<td>120 kgm^2</td>
</tr>
<tr>
<td>Spring stiffness</td>
<td>7 356 000 N/m</td>
</tr>
<tr>
<td>Pre-stressing force</td>
<td>300 000 N</td>
</tr>
<tr>
<td>Handrail post length</td>
<td>1.09 m</td>
</tr>
</tbody>
</table>

The resulting seven nonlinear ordinary differential equations of 2nd order can be linearized at the equilibrium point in case of small amplitudes guaranteed by active vibration control. Therefore, the nonlinear differential equations are derived into a nonlinear state space description, where the force generated by the two PMA \( F_{PM} = 4 \cdot AC_i \) is the input variable \( u \):

\[
x = f(x, u)
\]

For the 8-plate model, seven generalized coordinates and their first derivatives are selected as state vector \( x \).

\[
x = (x_1 \ x_2 \ \cdots \ x_{13} \ x_{14})^T = (\bar{z}_1 \ \bar{z}_2 \ \cdots \ \bar{z}_6 \ \bar{z}_7)^T = (\bar{z}_2 \ \bar{z}_3 \ \cdots \ \bar{z}_7 \ \bar{z}_8)^T
\]

The linear state space description (nodal form) is obtained by determining the Jacobian matrices \( (A, B) \) at the equilibrium point. Simulation and experiments of free vibrations without control are conducted to verify the dynamic behaviour of the model. In Table 2 the natural frequencies of the first three modes of the 8-plate model and of the measured bridge set-up are listed. The frequencies agree well with the measured frequencies.

Table 2. Natural frequencies of the 8-plate model and the Bridge set-up

<table>
<thead>
<tr>
<th>Mode</th>
<th>8-plate model</th>
<th>Bridge set-up</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. vertical symmetric</td>
<td>1.33 Hz</td>
<td>1.35 Hz</td>
</tr>
<tr>
<td>2. vertical asymmetric</td>
<td>2.54 Hz</td>
<td>2.76 Hz</td>
</tr>
<tr>
<td>3. vertical symmetric</td>
<td>3.85 Hz</td>
<td>4.08 Hz</td>
</tr>
</tbody>
</table>

To design a model based controller for specific modes it is more convenient to use the modal state space representation, Gawronski (2004). This representation is obtained from the already derived nodal state space representation by transformation using the modal matrix \( R \in \mathbb{R}^{14 \times 14} \). In the modal description all modes are decoupled.

\[
x_m = R^T \cdot x
\]

\[
A_m = R^{-1} A R \quad B_m = R^{-1} B \quad C_m = C R
\]

Where \( x_m \in \mathbb{R}^{14 \times 1} \) is the modal state vector, \( A_m \in \mathbb{R}^{14 \times 14} \) is the modal system matrix, \( B_m \in \mathbb{R}^{14 \times 1} \) is the modal input vector, \( C_m \in \mathbb{R}^{14 \times 14} \) is the modal output matrix and the nodal output matrix \( C \in \mathbb{R}^{14 \times 14} \) is the identity matrix.

3. KALMAN FILTER TO ESTIMATE MODAL STATES

To estimate the modal states a Kalman filter is employed, Lunze (2006). By measuring only the vertical displacement \( x_i \) in midspan the symmetrical modes can be observed. Therefore, only the symmetrical modes \( (SM) \) are extracted from the vectors / matrices of Eq. (4):

\[
A_m^{SM} = \text{diag}(A_m, A_m, A_m, A_m)
\]

\[
B_m^{SM} = [B_m \ B_m \ B_m \ B_m]^T
\]
\[
C^\text{SM}_m = [C_{m7} C_{m5} C_{m3} C_{m1}]
\]

(7)

Where \(A_{mi} \in \mathbb{R}^{2 \times 2}\), \(B_{mi} \in \mathbb{R}^{2 \times 1}\) and \(C_{mi} \in \mathbb{R}^{1 \times 2}\) are the components of mode \(i\).

The dynamics of the estimator are given by

\[
\dot{x}^\text{SM}_i = A^\text{SM}_i \cdot \dot{x}^\text{SM}_i + B^\text{SM}_i \cdot u + L \left[ y_s - C^\text{SM}_i \cdot \dot{x}^\text{SM}_i \right]
\]

(8)

\[
y_s = [x_q \ a_T \ s]_T
\]

(9)

where \(\dot{x}^\text{SM}_i\) are the estimated symmetric modal states. The measurement is \(y_s\) \((x_q\) is obtained by numerical differentiation) and \(\dot{y}_s\) is the estimation of this measurement based on \(\dot{x}^\text{SM}_i\). The matrix \(C_i\) selects the estimated elements \(\dot{x}_i\) and \(\dot{x}^\text{SM}_i\) out of the vector \(C^\text{SM}_i \cdot \dot{x}^\text{SM}_i\). The convergence of \(\dot{x}^\text{SM}_i \rightarrow \dot{x}^\text{SM}_i\) is determined by the observer gain \(L\). This gain is found by minimizing a quadratic cost function.

4. NONLINEAR MODEL OF THE ACTUATOR SYSTEM

The used PMA is developed and manufactured by the company Festo, Festo (2010). The pulling-only actuator consists of a flexible tube. By inflating the tube with compressed air it expands in radial direction and contracts in longitudinal direction. The force \(F_{\text{pma}}\) produced by the PMA \(i = [1, 2]\) depends on the pressure \(p_{\text{m}}\) inside the muscle and its contraction length \(s_{\text{m}}\). Detailed information about modelling of pneumatic actuators including mass flow, pressure dynamics and force characteristics are given in Hildebrandt et al. (2003) and Hildebrandt (2009). Input of the actuator system \(i\) is the reference position \(u_i\) of the proportional directional control valve. Output of the actuator system is the generated force \(F_{\text{pma}}\). In Bleicher (2011), the nonlinear actuator model is given by

\[
p_{\text{pma}} = f(p_{\text{pma}}, s_{\text{m}}) + g(p_{\text{pma}}, s_{\text{m}}) u_i
\]

(10)

where \(p_{\text{pma}}\) is the measurable system state, \(s_{\text{m}}\) is a time-variant measurable system variable, and \(\psi(q_s, b)\) is the flow rate function defined by Eq. (11). Further parameters are listed in Table 3.

\[
\psi(q_s, b) = \begin{cases} 
\sqrt{1 - \frac{q_s - b}{1 - b}}, & q_s \geq b \quad \text{(subcritical flow)} \\
1, & q_s < b \quad \text{(supercritical flow)}
\end{cases}
\]

(11)

Table 3. Parameters of the actuator system using a PMA type DMSP-40-356N and a valve type MYPE-5-1/4-420-B, Festo (2010)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Virtual piston area (A_i)</td>
<td>0.01022 m(^2)</td>
<td></td>
</tr>
<tr>
<td>Valve parameter (C_i)</td>
<td>6.25 litre/(s·bar)</td>
<td></td>
</tr>
<tr>
<td>Specific gas constant (R)</td>
<td>287 J/kgK</td>
<td></td>
</tr>
<tr>
<td>Gas temperature (T_i)</td>
<td>293 K</td>
<td></td>
</tr>
<tr>
<td>Valve parameter (b)</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td>Force constant (c_p)</td>
<td>0.09540 m</td>
<td></td>
</tr>
<tr>
<td>Force constant (c_i)</td>
<td>22130 kg/s(^2)</td>
<td></td>
</tr>
<tr>
<td>Force constant (c_s)</td>
<td>-493900 kg/s(^3)</td>
<td></td>
</tr>
<tr>
<td>Force constant (c_{\text{v}})</td>
<td>1849000 kg/s(^3)</td>
<td></td>
</tr>
<tr>
<td>Polytrophic exponent (n_p)</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>Ambient pressure (p_0)</td>
<td>101325 N/m(^2)</td>
<td></td>
</tr>
<tr>
<td>Supply pressure (p_i)</td>
<td>6 (p_0)</td>
<td></td>
</tr>
<tr>
<td>Volume constant (v_0)</td>
<td>0.43699 litre</td>
<td></td>
</tr>
<tr>
<td>Volume constant (v_1)</td>
<td>9.4724 litre/m</td>
<td></td>
</tr>
<tr>
<td>Volume constant (v_2)</td>
<td>-44.6014 litre/m(^2)</td>
<td></td>
</tr>
<tr>
<td>Isentropic exponent (\kappa)</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>Air density (\rho_0)</td>
<td>1.293 kg/m(^3)</td>
<td></td>
</tr>
</tbody>
</table>

5. FORCE CONTROL USING I/O LINEARISATION

Exact I/O linearization is applied to render the nonlinear actuator dynamics linear for forces in the range 0 to 3000 N and for \(-1 \leq u_i \leq 1\). The linearising and stabilising control law is given by

\[
u_i = -\left[ \frac{L^g h(p_{\text{pma}}, s_{\text{m}}) + \Delta s_{\text{m}}}{L^g h(p_{\text{pma}}, s_{\text{m}})} + a_0 F_{\text{m},\text{ref}} \right] \frac{a_0}{L^g h(p_{\text{pma}}, s_{\text{m}})} F_{\text{m},\text{ref}}
\]

(12)

where \(F_{\text{m},\text{ref}}\) is the measured force, \(F_{\text{m},\text{ref}}\) is the reference force, \(L, h\) and \(L^g, h\) are the \(L, h\) derivatives, \(\Delta s_{\text{m}}\) is the derivative of the time-variant measurable system variable and \(a_0\) is the control parameter to set the time constant of the resulting linear actuator dynamics which is given by

\[
G_{p_f}(s) = \frac{a_0}{s + a_0} = \frac{1}{1 + T_f \cdot s}.
\]

(16)

The control parameter is set to \(a_0 = 3.18\) Hz respectively \(T_f = 0.055\) s, Bleicher (2011). The controller behaves experimentally as expected as long as the supply pressure is constant also under load changes and if the sampling rate of the digital controller realisation is high enough.
6. VELOCITY FEEDBACK CONTROL TO REDUCE FIRST MODE VIBRATIONS

The concept to control first mode vibrations is based on a cascade control structure with a subsidiary force control and a superior velocity feedback control. The control gain \( K \) for positive feedback (negative plant gain) of the observed modal velocity was found by applying the root locus method (Dorf (2008)) assuming a non-saturated control signal (force) and negligible observer dynamics. Hence, closed-loop poles are defined as roots of the equation

\[ 1 + G_{FF}(s)G(s)K = 0 \]  

(17)

where \( G \) is the transfer function of the first mode and \( G_{FF} \) is the linear actuator dynamics given by Eq. (16).

The transfer function of the first mode (input: total force \( F_M = F_{M1} + F_{M2} \) produced by the two pneumatic muscles, output: modal velocity \( \dot{x}_{M1} \)) is derived using the Laplace transform of the modal state-space model:

\[ G(s) = \frac{-b_{m1}s}{s^2 + 2\zeta_1 \omega_1 s + \omega_1^2} \]  

(18)

Parameters of this transfer function are the natural frequency \( \omega_1 = 8.36 \text{ rad/s} \), the damping factor \( \zeta_1 = 0.0022 \) and \( b_{m1} = 0.0003422 \text{ rad/(s N)} \).

The optimal damping ratio \( \zeta_{opt} = 0.32 \) of the closed-loop system is achieved with the gain setting \( K = 17,500 \text{ Ns/rad} \).

Root locus analysis furthermore revealed that high gain velocity feedback cannot render the closed-loop system unstable. Due to the special pole-zero constellation (a zero at the origin and 3 poles) the conjugate complex poles will converge to \( -10 \pm j\infty \) for \( K \to \infty \) (at least theoretically for this ideal linear system model). Very large gains will lead to an undesirable increase in the natural frequency and a decrease of the damping ratio.

Up to now, no constraints in the control signal have been taken into account. Due to the nature of the chosen actuator, only limited unidirectional (positive) control forces can be applied. These forces push the bridge downwards. Simulations have shown that the practically applied gain should be doubled to obtain the above given optimal damping ratio. The Popov Criterion (Khalil (2001)) was successfully used to prove the stability of the closed-loop system formed by a static nonlinear gain (saturated velocity feedback law)

\[ F_{M,ref} = \begin{cases} \left( 2K \dot{x}_{M1} \right) & \text{for } 0 \leq 2K \dot{x}_{M1} \leq F_{M,max} \\ F_{M,max} & \text{for } 2K \dot{x}_{M1} > F_{M,max} \\ 0 & \text{for } \dot{x}_{M1} < 0 \end{cases} \]  

(19)

and the linear system

\[ \dot{x}_{M1}(s) = G(s)G_{FF}(s)F_{M,ref}(s) \]  

(20)

Having a maximal force of 3,000 N for one PMA it follows that \( F_{M,max} = 6,000 \text{ N} \).

The results of this control strategy are shown and discussed in Section 7.

Fig. 4. Control structure for experiment
7. RESULTS OF EXPERIMENT

For experiment the control structure was implemented in a real-time environment with a sampling frequency of 100 Hz. Therefore, a PC running a Linux operation system with the real-time extension RTAI (https://www.rtai.org/) was used. The software code was generated and compiled from Scilab / Scicos block diagrams. Communication to devices was provided by the HART Toolbox (http://hart.sourceforge.net).

The excitation signal for this experiment was a sinusoidal force signal in the first mode so that the pneumatic muscle actuators work first as exciter. Afterwards the vibrations are actively damped by the actuator. In Fig. 5 the reference force $F_{M1/2, ref}$ and the measured force $F_{M1, act}$ after excitation are plotted. It can be observed that the measured force follows the expected linear actuator dynamics $F_{M1/2, est, PT1}$ in good agreement. In Fig. 6 time histories of estimated modal velocity with velocity feedback control and without control are plotted. The control strategy efficiently reduces the first mode response. The uncontrolled response decays very slowly with logarithmic decrement $\Lambda_1, A \approx 0.034$. The controlled response decays about 26 times faster with $\Lambda_1, A, AVC \approx 0.740$.

Fig. 5. Reference force, actual force and estimated force by first order system (PT1)

The control performance under pedestrian excitation is shown in Fig. 7. One person is walking from one side to the other side with a step frequency of $\approx 1.35$ Hz that corresponds to the bridge’s first natural frequency. Without control a maximum acceleration amplitude of 6 m/s² occurs. With control the acceleration amplitude can be reduced by $\approx 89$ % to less than 0.6 m/s². No significant spillover effects in higher unsymmetrical modes could be observed.

Fig. 6. Modal velocity of the first mode w/o and with AVC

8. CONCLUSIONS

Active vibration control for a stress ribbon bridge with an extremely light pneumatic actuator was investigated. For this purpose, an analytical nonlinear bridge model was developed. By experiments it was confirmed that the linearized model represents the nonlinear behavior of the stress ribbon in good agreement for multi-modal motions. By using exact linearization methods the nonlinear dynamics of the pneumatic actuator were transformed into fast linear dynamic behavior. This step greatly simplifies the design for the higher level velocity feedback controller. The proposed cascaded control structure for active damping of the bridge mitigates first mode responses efficiently. To verify the model-based control designs full-scale experiments were conducted.

The used control strategy can be easily extended to multi-variable control, thus in further studies a multi-variable control system will be implemented to control asymmetric and higher symmetric modes. In addition, the efficiency of the controller reducing forced and random vibrations induced by a number of pedestrians will be investigated. Future analysis will also deal with the robustness of the controller under strongly varying masses by pedestrians.

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