State-based opacity of real-time automata

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Content

Background

- Opacity is a confidentiality property (firstly proposed by (Mazaré, 2004)) used to characterize information flow security, and has been widely used to describe all kinds of scenarios in security/privacy problems.

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- It describes whether a labeled (aka partially-observed) system can forbid an external intruder from making sure whether some secrets have been visited, given that the intruder knows complete knowledge of the system's structure but can only see outputs generated by the system.

A general framework for opacity

Run-based opacity (Bryans et al., 2008)

$$q_0 \xrightarrow{e_1} q_1 \xrightarrow{e_2} \cdots \xrightarrow{e_n} q_n \qquad (\forall \text{ secret run})$$

$$q'_0 \xrightarrow{e'_1} q'_1 \xrightarrow{e'_2} \cdots \xrightarrow{e'_m} q'_m \qquad (\exists \text{ non-secret run})$$

$$\text{s.t. } \ell(e_1 \dots e_n) = \ell(e'_1 \dots e'_m) \qquad (\text{the same label seq.})$$

Two special classes of opacity: I

Language-based (aka trace-based) opacity

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Verification results in untimed automata

- undecidable in labeled finite automata (LFAs) with ϵ-labeling functions (Bryans et al., 2008)
- decidable in EXPTIME in LFAs when secret languages and non-secrete languages are regular (Lin, 2011)

Two special classes of opacity: II

State-based opacity (specified according to the time when secrets visited)

$$q_0 \xrightarrow{e_1} q_1 \xrightarrow{e_2} \cdots \xrightarrow{e_i} q_i \xrightarrow{e_{i+1}} \cdots \xrightarrow{e_n} q_n \qquad (\forall \text{ secret state})$$

$$q'_0 \xrightarrow{e'_1} q'_1 \xrightarrow{e'_2} \cdots \xrightarrow{e'_j} q'_j \xrightarrow{e'_{j+1}} \cdots \xrightarrow{e'_m} q'_m \qquad (\exists \text{ non-secret state})$$

$$\text{s.t. } \ell(e_1 \dots e_i) = \ell(e'_1 \dots e'_j) =: \gamma_1$$

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$$q_0' \xrightarrow{e_1'} q_1' \xrightarrow{e_2'} \cdots \xrightarrow{e_j'} q_j' \xrightarrow{e_{j+1}'} \cdots \xrightarrow{e_m'} q_m' \qquad (\exists \text{ non-secret state})$$

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Verification results in untimed automata (plenty of)

Initial-state opacity (ISO) ($i = j = 0$), current-state opacity (CSO, $i = n, j = m$), infinite-step opacity (InfSO), and K-step opacity (KSO, $|\gamma_2| \leq K$) are PSPACE-complete in LFAs, and equivalent (Saboori and Hadjicostis, 2013) (Cassez, Dubreil, and Marchand, 2009) (Wu and Lafortune, 2013).

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- CSO is undecidable in time-deterministic event recording automata (Cassez, 2009).
- ISO is decidable in real-time automata (Wang, Zhan, and An, 2018).
- ISO, CSO, KSO, InfSO in real-time automata are decidable with 2-EXPTIME upper bounds (Zhang, 2021)

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- $\ell : E \to \Sigma \cup \{\epsilon\}$ is the labeling function.

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- A path is either ϵ or a sequence $q_0 \xrightarrow{e_1} q_1 \xrightarrow{e_2} \cdots \xrightarrow{e_n} q_n$, where $n \in \mathbb{Z}_+$, $(q_{i-1}, e_i, q_i) \in \Delta$ for all $i \in [\![1, n]\!]$.

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- The timed word of π is defined by $\tau(\pi) = (e_1, t_1')(e_2, t_2') \ldots (e_n, t_n')$, where $t_i' = \sum_{k=1}^{i} t_k$ for all $i \in [\![1, n]\!]$.
- The weight WT_π of π is defined by t_n'.

Example 1

Consider the RTA $\mathcal{A}_1$:

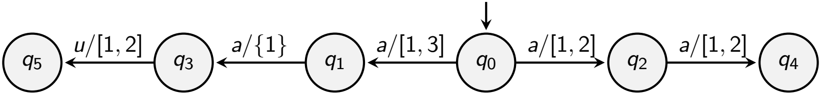

Figure 1: An RTA $\mathcal{A}_1$, q_0 is the initial state, a is an observable event, $\ell(a) = a$, u is unobservable, $\ell(u) = \epsilon$.

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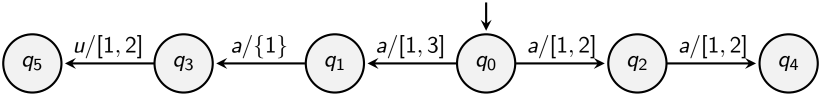

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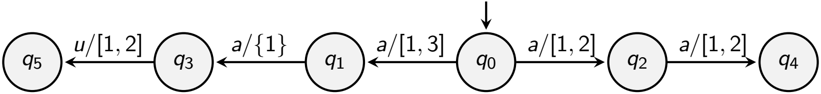

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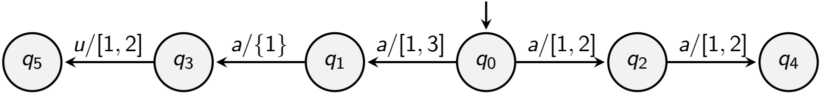

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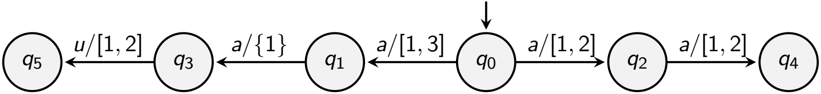

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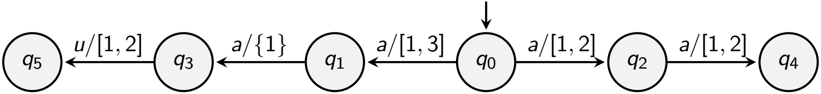

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\tau(\pi) &= (a,2)(a,3)(u,4), && \text{(timed word)} \\
\mathrm{WT}_\pi &= 4, && \text{(weight)} \\
\ell(\tau(\pi)) &= (a,2)(a,3). && \text{(timed label seq.)}
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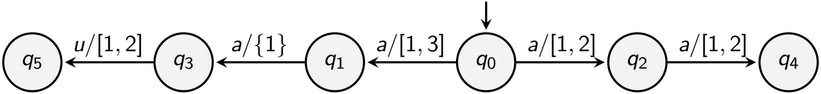
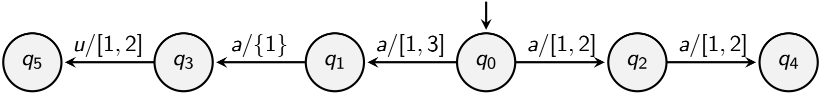
- $\mathrm{interm}(\gamma_1, \gamma_2) = \{q \in Q | (\exists \text{ runs } \pi_1, \pi_2)[(\mathrm{init}(\pi_1) \in Q_0) \wedge (\mathrm{last}(\pi_1) = \mathrm{init}(\pi_2) = q) \wedge (\ell(\tau(\pi_1)) = \gamma_1) \wedge (\ell(\tau(\pi_1\pi_2)) = \gamma_1\gamma_2) \wedge (\mathrm{WT}_{\pi_1} = \mathrm{last}_R(\gamma_1)) \wedge (\mathrm{WT}_{\pi_2} = \mathrm{last}_R(\gamma_2) - \mathrm{last}_R(\gamma_1))]\}$: the set of states $\mathcal{A}$ can be in when $\mathcal{A}$ has just generated timed label seq. γ_1, given that the current timed label seq. is $\gamma_1\gamma_2 \in (\Sigma \times \mathbb{R}_{\geq 0})^*$.

$$\overbrace{\underbrace{q_0 \xrightarrow{E^* E_o} q_1}_{\gamma} \underbrace{\longrightarrow q}_{\text{unobs.}}}^{\gamma} \qquad \overbrace{\underbrace{\underbrace{q_0 \xrightarrow{E^* E_o} q_1}_{\gamma_1} \underbrace{\longrightarrow q}_{\text{inst. unobs.}}}_{\gamma_1} \xrightarrow{E^* E_o} \underbrace{q_2 \longrightarrow q_3}_{\text{inst. unobs.}}}^{\gamma_1\gamma_2}$$

Example 2 (cont. $\mathcal{A}_1$)

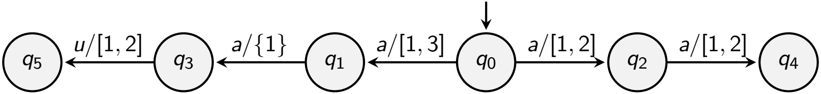


Example 2 (cont. $\mathcal{A}_1$)



$$\mathrm{last}([(a,2)]) = \{q_1, q_2\},$$

Example 2 (cont. $\mathcal{A}_1$)



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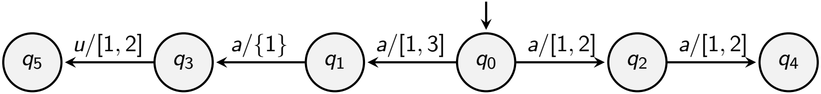

$$\begin{aligned}
\mathrm{last}([(a,2)]) &= \{q_1, q_2\},\\
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\end{aligned}$$

Current-state estimate

For $\mathcal{A}$, $x \subset Q$, and $\gamma \in (\Sigma \times \mathbb{R}_{\geq 0})^*$, the current-state estimate is

$$\mathcal{M}(\mathcal{A}, \gamma | x) := \{q \in Q | (\exists q_0 \in x)(\exists n \in \mathbb{N})(\exists m \in \mathbb{N}) \left(\exists \text{ a run } \pi = q_0 \xrightarrow{e_1/t_1} \cdots \xrightarrow{e_n/t_n} q_n \xrightarrow{e_{n+1}/0} \cdots \xrightarrow{e_{n+m}/0} q\right) [(e_n \in E_o) \wedge (e_{n+1} \dots e_{n+m} \in (E_{uo})^*) \wedge \ell(\tau(\pi)) = \gamma]\}.$$

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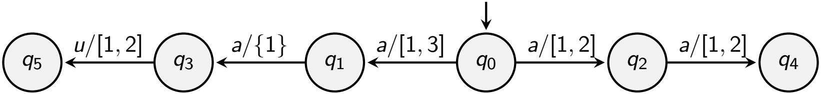
$\mathcal{M}(\mathcal{A},\gamma)$ denotes the set of states $\mathcal{A}$ can be in when γ has been generated.

$\mathcal{M}(\mathcal{A},\gamma|Q_0) =: \mathcal{M}(\mathcal{A},\gamma) \subset \text{last}([\gamma])$ for $\gamma \in (\Sigma \times \mathbb{R}_{\geq 0})^*$.

Example 3 (cont. $\mathcal{A}_1$)



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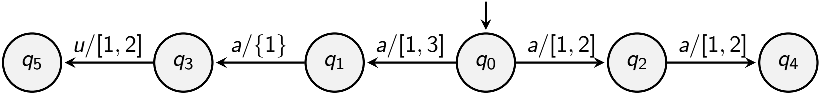

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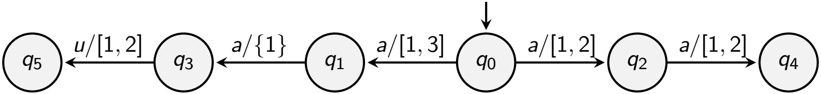

$$\mathrm{last}([(a,2)]) = \{q_1, q_2\},$$
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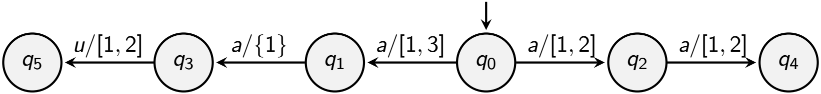
Example 3 (cont. $\mathcal{A}_1$)



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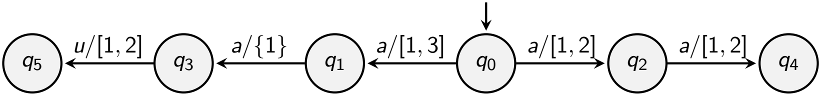

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\end{aligned}
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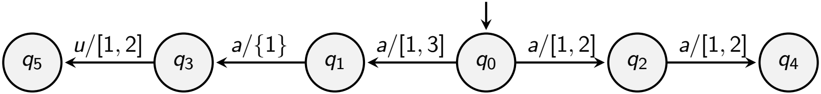

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\mathcal{M}(\mathcal{A}_1, (a,2)(a,3)) &\subsetneq \mathrm{last}([(a,2)(a,3)]).
\end{aligned}
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Content

Language generated by RTA $\mathcal{A}$: $\mathcal{L}(\mathcal{A}) = \{\gamma \in (\Sigma \times \mathbb{R}_{\geq 0})^* | \mathcal{M}(\mathcal{A}, \gamma) \neq \emptyset\}$

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An RTA $\mathcal{A}$ is called initial-state opaque (ISO) w.r.t. Q_S if for every $\gamma \in \mathcal{L}(\mathcal{A})$, $\mathrm{init}([\gamma]) \not\subset Q_S$.

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Definition 5 (CSO)

An RTA $\mathcal{A}$ is called current-state opaque (CSO) w.r.t. Q_S if for every $\gamma \in \mathcal{L}(\mathcal{A})$, in $\mathcal{M}(\mathcal{A}, \gamma)$ there exists at least one non-eventually-secret state.

Definition 6

A state q of an RTA $\mathcal{A}$ is called eventually secret if either (1) q is secret or (2) there is an unobservable path starting from q and along each of such paths at least one secret state will be visited.

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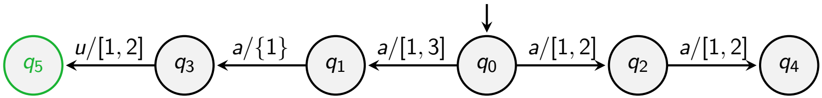
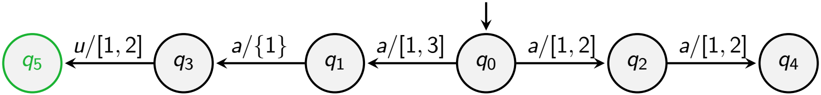
Example 7 (cont. $\mathcal{A}_1$)



Let $Q_S = \{q_5\}$. Then q_3 is eventually secret because of the unique unobservable path $q_3 \xrightarrow{u} q_5$ (with q_5 dead, i.e., no transition starts at q_5).

Example 8 (cont. $\mathcal{A}_1$)



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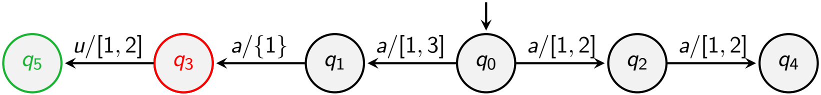
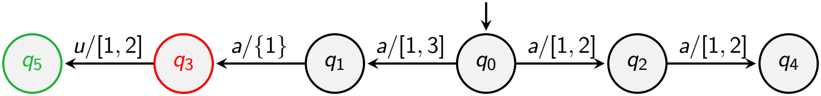


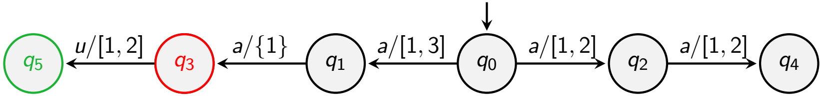
Let $Q_S = \{q_5\}$. In Example 7, we have shown q_3 is eventually secret.

Example 8 (cont. $\mathcal{A}_1$)



Let $Q_S = \{q_5\}$. In Example 7, we have shown q_3 is eventually secret. In addition, none of q_1, q_0, q_2, q_4 is eventually secret.

Example 8 (cont. $\mathcal{A}_1$)



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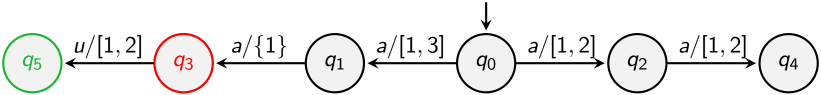

Let $Q_S = \{q_5\}$. In Example 7, we have shown q_3 is eventually secret. In addition, none of q_1, q_0, q_2, q_4 is eventually secret. We have $\mathcal{M}(\mathcal{A}_1, (a,3)(a,4)) = \{q_3\}$ which does not contain any non-eventually-secret state. Hence $\mathcal{A}_1$ is not CSO w.r.t. $\{q_5\}$.

Definition 9 (InfSO and KSO)

An RTA $\mathcal{A}$ is called infinite-step opaque (InfSO) w.r.t. Q_S if for all $\gamma_1\gamma_2 \in \mathcal{L}(\mathcal{A})$ such that $1 \le |\gamma_2|$, $\mathrm{interm}(\gamma_1, \gamma_2)$ contains at least one non-secret state q.

$$\overbrace{\underbrace{q_0 \xrightarrow{E^* E_o}}_{\gamma_1} \underbrace{q' \longrightarrow q}_{\text{inst. unobs.}} \underbrace{\xrightarrow{E^* E_o}}_{} \underbrace{q'' \longrightarrow q'''}_{\text{inst. unobs.}}}^{\gamma_1\gamma_2} \qquad (*)$$

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When observing $\gamma_1\gamma_2$ with $1 \leq |\gamma_2|$, one cannot make sure whether the state when γ_1 has just been generated is secret.

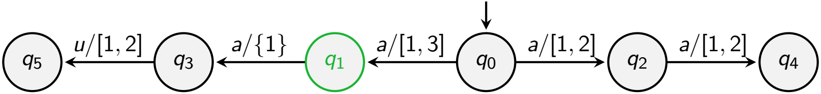
Definition 9 (InfSO and KSO)

An RTA $\mathcal{A}$ is called infinite-step opaque (InfSO) (K-step opaque (KSO)) w.r.t. Q_S if for all $\gamma_1\gamma_2 \in \mathcal{L}(\mathcal{A})$ such that $1 \le |\gamma_2| (\le K)$, interm(γ_1, γ_2) contains at least one non-secret state q.

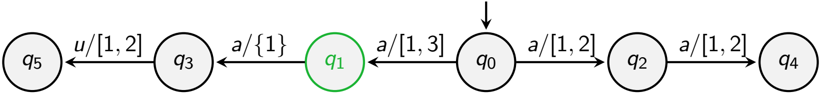
$$\overbrace{\underbrace{q_0 \xrightarrow{E^* E_o}}_{\gamma_1} \underbrace{q' \longrightarrow}_{\text{inst. unobs.}} q \underbrace{\xrightarrow{\qquad E^* E_o \qquad}}_{(\le K \text{ obs. events})} \underbrace{q'' \longrightarrow q'''}_{\text{inst. unobs.}}}^{\gamma_1\gamma_2} \qquad (*)$$

When observing $\gamma_1\gamma_2$ with $1 \le |\gamma_2| (\le K)$, one cannot make sure whether the state when γ_1 has just been generated is secret.

Example 10 (cont. $\mathcal{A}_1$)



Example 10 (cont. $\mathcal{A}_1$)



For $(a,3)(a,4)$, we only have

$$\overbrace{\underbrace{q_0 \xrightarrow{a/3}}_{\gamma_1=(a,3)} \underbrace{q_1 \xrightarrow{\quad\epsilon\quad} q_1}_{\text{unobs. secret}}}^{\gamma_1\gamma_2=(a,3)(a,4)} \xrightarrow{a/1} q_3 \Longrightarrow \mathrm{interm}(\mathcal{A}_1,(a,3),(a,4)) = \{q_1\}$$

which violates InfSO, i.e., $\mathcal{A}_1$ is not InfSO w.r.t. $\{q_1\}$.

Definition 11

For an RTA $\mathcal{A}$, we define its pre-observer as a deterministic automaton

$$\mathcal{A}_{\text{obs}}^{\text{pre}} = (X, \Sigma \times \mathbb{R}_{\geq 0}, x_0, \delta_{\text{obs}}^{\text{pre}}), \tag{1}$$

where $X \subset 2^Q \setminus \{\emptyset\}$ is the state set, $\Sigma \times \mathbb{R}_{\geq 0}$ the (infinite) alphabet, $x_0 = \mathcal{M}(\mathcal{A}, \epsilon) \in X$ the unique initial state, $\delta_{\text{obs}}^{\text{pre}} \subset X \times (\Sigma \times \mathbb{R}_{\geq 0}) \times X$ the transition relation: for all $x, x' \in X$ and $(\sigma, t) \in \Sigma \times \mathbb{R}_{\geq 0}$, $(x, (\sigma, t), x') \in \delta_{\text{obs}}^{\text{pre}}$ iff $x' = \mathcal{M}(\mathcal{A}, (\sigma, t)|x)$. For all $x \subset Q$ different from x_0, $x \in X$ iff there is $\gamma \in (\Sigma \times \mathbb{R}_{\geq 0})^+$ such that $x = \mathcal{M}(\mathcal{A}, \gamma)$.

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- After $\delta_{\text{obs}}^{\text{pre}}$ is recursively extended to $\delta_{\text{obs}}^{\text{pre}} \subset X \times (\Sigma \times \mathbb{R}_{\geq 0})^* \times X$, one has for all $x \in X$ and $(\sigma_1, t_1) \ldots (\sigma_n, t_n) =: \gamma \in (\Sigma \times \mathbb{R}_{\geq 0})^+$, $(x_0, \gamma, x) \in \delta_{\text{obs}}^{\text{pre}}$ iff $\mathcal{M}(\mathcal{A}, \tau(\gamma)) = x$, where $\tau(\gamma) = (\sigma_1, t_1)(\sigma_1, t_1 + t_2) \ldots (\sigma_n, t_1 + \cdots + t_n)$.

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- Alphabet $\Sigma \times \mathbb{R}_{\geq 0}$ is not finite, one cannot compute the whole $\mathcal{A}_{\text{obs}}^{\text{pre}}$. Next, we define observer $\mathcal{A}_{\text{obs}}$ as a finite sub-automaton of $\mathcal{A}_{\text{obs}}^{\text{pre}}$.

Definition 12

For an RTA $\mathcal{A}$, consider its pre-observer $\mathcal{A}_{\mathrm{obs}}^{\mathrm{pre}} = (X, \Sigma \times \mathbb{R}_{\geq 0}, x_0, \delta_{\mathrm{obs}}^{\mathrm{pre}})$, we define its observer as a finite automaton

$$\mathcal{A}_{\mathrm{obs}} = (X, \Sigma_{\mathrm{obs}}, x_0, \delta_{\mathrm{obs}}), \tag{2}$$

where Σ_{obs} (resp., δ_{obs}) is a finite subset of $\Sigma \times \mathbb{Q}_{\geq 0}$ (resp., $\delta_{\mathrm{obs}}^{\mathrm{pre}}$), such that if there exists a transition from $x \in X$ to $x' \in X$ in $\delta_{\mathrm{obs}}^{\mathrm{pre}}$ then at least one such transition belongs to δ_{obs}.

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Remark 1

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Remark 1

- *For an RTA $\mathcal{A}$, it may have more than one observer, because Σ_{obs} may not be unique; but X and x_0 must be unique.*

Definition 12

For an RTA $\mathcal{A}$, consider its pre-observer $\mathcal{A}_{\text{obs}}^{\text{pre}} = (X, \Sigma \times \mathbb{R}_{\geq 0}, x_0, \delta_{\text{obs}}^{\text{pre}})$, we define its observer as a finite automaton

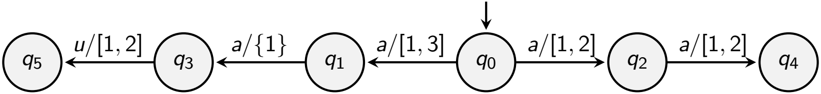
$$\mathcal{A}_{\text{obs}} = (X, \Sigma_{\text{obs}}, x_0, \delta_{\text{obs}}), \tag{2}$$

where Σ_{obs} (resp., δ_{obs}) is a finite subset of $\Sigma \times \mathbb{Q}_{\geq 0}$ (resp., $\delta_{\text{obs}}^{\text{pre}}$), such that if there exists a transition from $x \in X$ to $x' \in X$ in $\delta_{\text{obs}}^{\text{pre}}$ then at least one such transition belongs to δ_{obs}.

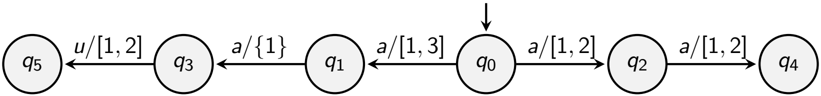
Remark 1

- *For an RTA $\mathcal{A}$, it may have more than one observer, because Σ_{obs} may not be unique; but X and x_0 must be unique.*
- *For a labeled finite automaton, it has a unique observer, which is actually the powerset construction used for determinizing the automaton.*

Example 13 (cont. $\mathcal{A}_1$)



Example 13 (cont. $\mathcal{A}_1$)



One of its observers is

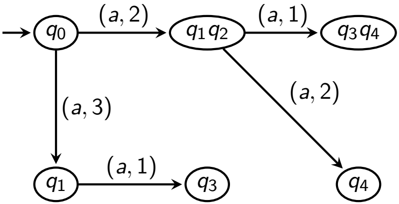


Figure 2: $\mathcal{A}_{1\,\mathrm{obs}}$.

Theorem 14

An RTA $\mathcal{A}$ is CSO w.r.t. Q_S iff in observer $\mathcal{A}_{\mathrm{obs}}$, every reachable state x contains at least one non-eventually-secret state of $\mathcal{A}$.

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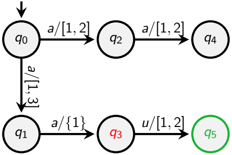
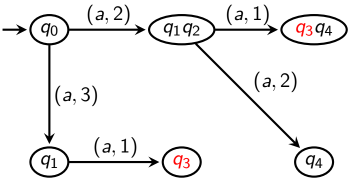
An RTA $\mathcal{A}$ is CSO w.r.t. Q_S iff in observer $\mathcal{A}_{\mathrm{obs}}$, every reachable state x contains at least one non-eventually-secret state of $\mathcal{A}$.

Example 15 (cont. $\mathcal{A}_1$)



$\mathcal{A}_1$.



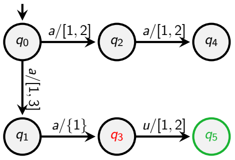
$\mathcal{A}_{1\,\mathrm{obs}}$.

Let $Q_S = \{q_5\}$, so the eventually secret states are q_3 and q_5.

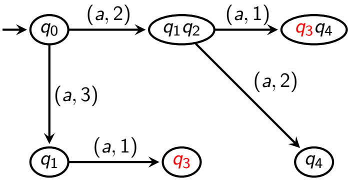
Theorem 14

An RTA $\mathcal{A}$ is CSO w.r.t. Q_S iff in observer $\mathcal{A}_{\mathrm{obs}}$, every reachable state x contains at least one non-eventually-secret state of $\mathcal{A}$.

Example 15 (cont. $\mathcal{A}_1$)



$\mathcal{A}_1$.



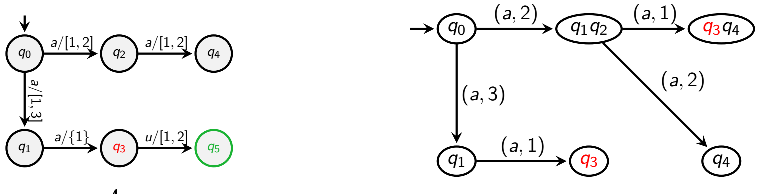
$\mathcal{A}_{1\,\mathrm{obs}}$.

Let $Q_S = \{q_5\}$, so the eventually secret states are q_3 and q_5. In $\mathcal{A}_{1\,\mathrm{obs}}$, there is a reachable state $\{q_3\}$ which only contains eventually secret states, then $\mathcal{A}_1$ is not CSO w.r.t. $\{q_5\}$.

Theorem 14

An RTA $\mathcal{A}$ is CSO w.r.t. Q_S iff in observer $\mathcal{A}_{\text{obs}}$, every reachable state x contains at least one non-eventually-secret state of $\mathcal{A}$.

Example 15 (cont. $\mathcal{A}_1$)



$\mathcal{A}_1$. $\mathcal{A}_{1\,\text{obs}}$.

Let $Q_S = \{q_5\}$, so the eventually secret states are q_3 and q_5. In $\mathcal{A}_{1\,\text{obs}}$, there is a reachable state $\{q_3\}$ which only contains eventually secret states, then $\mathcal{A}_1$ is not CSO w.r.t. $\{q_5\}$.

For an RTA $\mathcal{A}$, its observer $\mathcal{A}_{\text{obs}}$ can be computed in 2-EXPTIME.

Content

Results

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- Notions of observer and reverse observer of RTAs

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- Notions of state-based opacity in real-time automata (RTAs)
- Notions of observer and reverse observer of RTAs
- Verification of state-based opacity with complexity upper bounds

Thank you for your attention!

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