Nonlinear Joint-Angle Feedback Control of Electrically Stimulated and $\lambda$-Controlled Antagonistic Muscle Pairs

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Abstract—In order to control human limb movements in neuro-prosthetic systems, a nonlinear, model-based control strategy for torque generation by antagonistic muscle pairs is presented. The controller based on exact linearization methods enables the tracking of reference joint torque profiles and the generation of pre-defined muscular co-contractions. The controller adjusts the desired recruitment levels $\lambda$ of both muscles that are controlled by two underlying $\lambda$-Controllers automatically compensating muscular fatigue. Estimates on $\lambda$ are obtained from electrical stimulation evoked Electromyography (EMG)-measurements. Short-term increases of the muscular variable constraints which result from the fact that only positive muscular contractions can occur. The linearization-based controller can serve as underlying controller in cascaded control schemes for the joint-angle or velocity. The feasibility of the proposed approach was demonstrated in a simulation study as well as by a joint-angle control experiment for a healthy subject. Concluding, the authors present that the proposed approach has a great potential for the future control of artificial limb movements also because of the possibility of modulating the system stiffness by controlled co-contractions of the muscles.

I. INTRODUCTION

Neuro-prosthetic systems aim to induce limb movements by muscular contractions artificially caused by functional electrical stimulation (FES) [8]. Such systems are generally controlled by feedforward or feedback control strategies which modulate the intensity of electrical pulses in terms of charge, current amplitude, pulselength or frequency to achieve a certain goal, e.g., tracking a reference trajectory for a joint angle (e.g. [10]).

For joint-angle control, often antagonistic muscle pairs are available for exploitation of actuation, e.g., for the elbow-/knee-joint angle, or for the horizontal shoulder adduction/abduction. The latter will be considered in detail within this contribution. This control problem arises for example in the context of restoring arm function in tetraplegic patients with a high lesion of the spinal cord.

It has been shown that for natural limb movements the involved muscle pairs are not limited to be actuators that solely induce pulling forces into their pre-defined directions. Instead, an important property for the natural control of posture and movement is the ability to allow the modulation of the mechanical impedances due to co-contractions. Naturally, they are modulated by the central nervous system [5].

For FES, more than twenty years ago it was demonstrated, that the involvement of co-contractions can significantly increase the performance of control systems using FES for actuation [3]. The positive effects of co-contractions are already exploited in control of FES (e.g. for compensation of tremor [2], ...).

An important component of muscle modeling and control is the nonlinear, spatial recruitment curve which describes the number of motor units in a muscle activated by FES. This static function, depending on the stimulation intensity, usually includes a threshold at a relative high stimulation level. In order to also control small co-contraction forces, this parameter has to be known exactly for both muscles, otherwise the control system has no information whether the muscle is actually contracted or whether the actuation variable is just acting below the threshold without any influence. Because of muscular fatigue, this parameter is variable in time and hence an on-line identification would be required. However, this is rather difficult when only the summed torque by both muscles is available indirectly through joint-angle measurements.

A promising approach suitable to overcome the problem of inexact known and time-variant thresholds as well as other muscular properties like fatigue and hysteresis, was presented in an earlier publications [6] and [7]. The aim is to enforce a linear behavior for the nonlinear recruitment curve by feedback ($\lambda$-Control) of an estimate of the muscular recruitment $\lambda$ (amount of activated motor neurons). The stimulation intensity is adjusted such that a reference recruitment level $r_2$ is tracked. Controllers for neuro-prosthetic systems would then use this reference for the muscular recruitment instead of the stimulation intensity as their actuation variable(s).

In this work, a strategy for modulation of a desired accelerating torque along with a co-contraction torque in form of a linearizing controller is presented. This controller acts on top of two $\lambda$-Control loops for each muscle respectively. The linearizing controller is based on system inversion for time discrete systems. Since the signals for the actuation of each muscle are constraint to be positive, an extension of the linearizing controller is investigated that guarantees exact inversion with regard to the accelerating torque at the cost of a temporally increased co-contraction torque. For demonstrating this concept, a linear joint-angle controller acting on top of the linearizing controller is designed.

Sec. II describes the experimental set-up. A model for control design is introduced in Sec. III followed by the linearization controller in Sec. IV. In Sec. V a linear feedback controller for controlling the joint angle is designed for demon-
strating the usefulness of the underlying linearizing controller. Results are presented in Sec. VI and final conclusions are given in Sec. VII.

II. CONTROL SCHEME & EXPERIMENTAL SET-UP

A. Experimental set-up

The used set-up is shown in Fig. 1 and consists of an passive exoskeleton (ARMEO, Hocoma AG, Switzerland) for weight compensation with angle sensors, a stimulation system (REHASTIM, Hasomed GmbH, Germany), a 24-bit EMG-amplifier (PHYSIOSENSE, developed at TU Berlin), an inertial motion unit (IMU) (RAZORIMU 9DoF, Sparkfun, United Kingdom) and a PC running Linux with RT-Preemption-Patch. All devices are connected to the PC through USB interfaces.

The subject’s arm is placed inside the exoskeleton as shown and the arm movement is limited to horizontal shoulder abduction and adduction. The corresponding angle $\vartheta$ is defined as shown in Fig. 1 whereas $\vartheta = 0$ describes the rest position of the arm. This angle and its first time derivative are determined by fusing the outputs of the exoskeleton angle sensors and the IMU information.

Current-controlled stimulation impulses are applied through self-adhesive hydroygel electrodes at a stimulation frequency of 28 Hz. The anterior part of the deltoid muscle is stimulated to cause horizontal shoulder adduction. Horizontal abduction is produced by stimulation of the posterior part of the deltoid muscle. Signals related to adduction and abduction will be denoted by the indices 1 and 2 respectively. For EMG measurement at the stimulated muscles, smaller AgCl electrodes are placed between each pair of stimulations electrodes. The FES evoked EMG (eEMG) is recorded at a sampling rate of 4kHz.

For both muscles, the eEMG is filtered as described in [6] to estimate the states $\lambda_i$, $i = 1, 2$, of muscle recruitment. Two $\lambda$-controllers, also introduced in [6], automatically adjust the stimulation intensity (pulse charge) in order to generate the desired recruitment levels $r_{\lambda_i}$, $i = 1, 2$.

The open source real-time dynamic block simulation system OPENRTDYNAMICS (http://openrtdynamics.sf.net) is used for implementation of all time critical components and provides a network communication to a QT4-GUI. The design of the control system is carried out with help of the program system SCILAB (http://www.scilab.org).

III. NEURO-MUSCULOSKELETAL MODEL

To describe the joint angle dynamics with FES actuation, a Hill-type muscle model [4] succeeded by a second order mechanical system is assumed. This model along with methods for parameter estimation is described in [11].

A. Muscle model including $\lambda$-Controller

The muscular model part, which also includes the closed loop of the $\lambda$-Controller as shown in Fig. 2, describes the generated torque $T_i$ for each muscle $i$ respectively. Because the muscles are $\lambda$-controlled, the inputs are the references for the recruitment levels.

Internally, the $\lambda$-Controller systems adjust the stimulation intensities $v_i$ for each muscle $i$ such that the muscular recruitment levels $\lambda_i$ follow their references $r_{\lambda_i}$. Hereby, integral controllers are used to control the one step delayed output of the recruitment functions $r_{\lambda_i}$ (the estimated muscular recruitment $\hat{\lambda}$). These functions commonly consist of a threshold, a saturation and an approximatively linear behavior inbetween. For the ongoing control design, the resulting closed loop systems are assumed to be linear, whereby the reference to output behaviors are then described by the transfer functions $T_{\lambda_i}$. Each $\lambda$-Controller gain is determined by obtaining a linear model of $r_{\lambda_i}$ and a subsequent pole placement design as described in [6]. The ability to compensate for muscular fatigue and time variant thresholds in $r_{\lambda_i}$ was shown in [7].

The calcium dynamics $G_m$ models the dependency of the muscle activation $\sigma_i$ on $\lambda_i$. The recruitment $\lambda_i$, its reference $r_{\lambda_i}$, the stimulation intensity $v_i$ as well as the muscular activation $\sigma_i$ are constrained to be non-negative. The calcium dynamics can be approximated by a continuous-time transfer function of first order without zeros and a time delay $T_{ca}$. The transfer function is time discretized using Euler discretization:

$$G_m(q^{-1}) = 1/(sT_{ca} + 1)|_{s=(1-q^{-1})/T_a}.$$

Here, $q^{-1}$ is the backward shift operator and $T_a = 36$ ms is the sampling period. The approximative time constant $T_{ca} = 0.04$ s is chosen with reference to [9]. The time delay of the calcium dynamics is approximated by a delay of two sampling steps ($q^{-2}$), which matches to measured data quite well. The closed loop systems $T_{\lambda_i}$ are discrete-time first order system without dead time and zeros [6].

Finally, the muscular torque $T_i$ is then modeled by the multiplication of a function $f_i$ with $\sigma_i$. The function $f_i$ linearly depends on the joint angle and is modeled by:

$$f_i(k) = k_{v,i}(\dot{\vartheta}_{max,i} - \dot{\vartheta}(k)).$$

The parameters $\dot{\vartheta}_{max,i}$ represent the boundaries of the operational angular space $\dot{\vartheta} \in [\dot{\vartheta}_{max,2}, \dot{\vartheta}_{max,1}]$ and describe
reachable joint angles. The gains $k_{\lambda,1}$ and $k_{\lambda,2}$ are strictly positive, which results in $f_1 \geq 0$ and $f_2 \leq 0$ and further in $T_1 \geq 0$ and $T_2 \leq 0$ respectively because of $\sigma_1 \geq 0$.

![Diagram of a controller](image)

**Fig. 2** - The model for one $\lambda$-Controlled muscle path. Given a reference $r_\lambda$ for the muscular recruitment $\lambda$, the controller adjusts the stimulation intensity $v$. The nonlinear recruitment function $rc_i$ describes the recruitment. The resulting closed loop $T_\lambda$ has an approximately linear dynamics. The calcium dynamics $G_m$ models the dependency of the muscular activation $\sigma_i$ on $\lambda$. The muscular torque $T_i$ is then modeled by the multiplication of the angle-dependent function $f_\lambda$ with $\sigma_i$.

For the linearizing controller designed later on, the slightly more conservative constraints $f_1 > 0$ and $f_2 < 0$ must be applied. Therefore, the valid range for the joint angle is reduced to $\hat{\vartheta} \in [\hat{\vartheta}_{\text{max},2}, \hat{\vartheta}_{\text{max},1}]$.

The total muscle model is then described by

$$
T_i(k) = k_{\lambda,i}(\hat{\vartheta}_{\text{max},i} - \hat{\vartheta}(k)) \cdot \frac{q^{-2}(1 - a)}{f_i(k)} \cdot \frac{1 - b_i}{1 - bq^{-1}} \cdot \frac{1 - b_iq^{-1}}{q^{-2}G_m(q^{-1})} r_{\lambda,i} \tag{1}
$$

where $k$ is the sampling index. The parameters $a \in [0,1]$ and $b_i \in [0,1]$ lead to an asymptotically stable system.

**B. Resulting acceleration and co-contraction torque**

For describing a torque $T$ which leads to an acceleration of the mechanical system (arm) and to a co-contractive torque $T_{cc}$, which describes the amount of co-contracting torques in an antagonistic muscle pair, a transformation between both muscular induced torques $T_i$ and the resulting torques $T$ and $T_{cc}$ is introduced.

The accelerating torque $T$ is given by the operation

$$
T(T_1, T_2) := T_1 + T_2, \quad T_1 \geq 0, \quad T_2 \leq 0. \tag{2}
$$

The co-contraction torque $T_{cc} \geq 0$ is obtained by the following operator:

$$
T_{cc}(T_1, T_2) := \min(T_1, -T_2) \tag{3}
$$

The inverse transformation operator $T_i^I$ is then

$$
T_i^I(T_{cc}, T) := \begin{cases} 
  T_{cc} + T & T \geq 0 \\
  T_{cc} & T < 0 
\end{cases} \tag{4}
$$

and analog

$$
T_i^I(T_{cc}, T) := \begin{cases} 
- T_{cc} & T \geq 0 \\
- T_{cc} + T & T < 0 
\end{cases} \tag{5}
$$

The torques $T$ and $T_{cc}$ are then described by

$$
T(k) = T(T_i(k), T_2(k)), \quad T_{cc}(k) = T_{cc}(T_1(k), T_2(k)).
$$

**C. Mechanical model**

The input to the mechanical model are the resulting torques $T$ and $T_{cc}$. Because of the one dimensional movement, a second order system including a viscous friction $R_u$ and the elasticity $K_m$ is used. The differential equation in terms of the joint angle $\vartheta$ is

$$
\ddot{\vartheta} = \frac{1}{J} [T - R_m(T_{cc})\dot{\vartheta} - K_m(T_{cc})\dot{\vartheta}] \tag{6}
$$

A modulation of the mechanical impedance by the co-contraction torque $T_{cc}$ is assumed. In this case, the parameters for the viscous friction $R_u$ and the elasticity $K_m$ depend on $T_{cc}$. The design of a model for these functions would go beyond the scope of this paper and forms future research topics. Therefore, for controller design this modulation of the system properties is neglected and only the constants obtained through system identification for zero co-contraction are used.

The complete model including the $\lambda$-controlled muscles and the mechanical system is presented in Fig. 3. The mechanical model as described in [11], also accounts for a static friction term, which is neglected in this work.

**IV. LINEARIZING CONTROLLER**

The multiplication inside the muscle models with the internal feedback signal $f_i$ leads to a non-linear system behavior. Since the mechanical system is already linear, only the muscular part, which contains the multiplication, is linearized. Therefore, a state space representation of (1) is derived with $\overline{\lambda}_i = q^{-1}\lambda_i$:

$$
\overline{\lambda}_i(k + 1) = b_i\overline{\lambda}_i(k) + (1 - b_i)r_{\lambda,i} \tag{7}
$$

$$
\sigma_i(k + 1) = a\sigma_i(k) + (1 - a_i)[b_i\lambda_i(k) + (1 - b_i)r_{\lambda,i}] \tag{8}
$$

$$
T_i(k) = \sigma_i(k)f_i(k). \tag{9}
$$

For describing the muscular torque induced by each muscle $i$ with two time steps delay, two functions $T_i, i = 1, 2$, are introduced that depend on the actuation variables $r_{\lambda,i}$:

$$
T_i(r_{\lambda,i}) := f_i(k + 2)[a\sigma_i(k + 1) + (1 - a)[b_i\lambda_i(k) + (1 - b_i)r_{\lambda,i}]] \tag{10}
$$

The modeled torque is then described by $T_i(k + 2) = T_i(r_{\lambda,i}(k))$. For the control law, two functions

$$
R_{\lambda,i}(u_i) := \frac{1}{1 - b_i} \left[ - b_i\lambda_i(k) + \frac{1}{1 - a_i} \left[ - a\sigma_i(k + 1) + \frac{1}{f_i(k)}u_i \right] \right] \tag{11}
$$

are introduced with $i = 1, 2$. By assigning them to the actuation variables $r_{\lambda,i} = R_{\lambda,i}(u_i(k))$ the system (1) is exactly linearized. Eq. (11) depends on the future internal mechanical feedback variable $f_i(k + 2)$ as well as on the state variables $\sigma_i(k + 1)$ and $\lambda_i(k)$. The states are calculated by two internal models implementing (7) and (8) for $i \in 1, 2$, whereby the actuation variables $r_{\lambda,i}(k)$ are applied to this internal system. An estimation of $f_i(k + 2)$ is used instead of the real signal, that is calculated by a two-step ahead prediction (see
Fig. 3 - Neuro-musculoskeletal model used for controller design.

Fig. 4 - An extension is applied to the linearizing controller in the presence of constrained actuation by modification of the signals \( u_1 \) and \( u_2 \). The calculated actuation variable of the linearization controller is applied to the plant, which additionally includes the constraints on the actuation variables \( r_{\lambda_i} \).

Because of the dynamics of the muscle contraction and the \( \lambda \)-Control loop, the muscular activations \( \sigma_i(k+1) \) are not necessarily zero for zero actuation. Instead, the smallest muscular activation possible is achieved by choosing \( r_{\lambda_i}(k) = 0 \) as negative actuation variables are not possible. The torques produced by each muscle in this case are called minimum torques:

\[
T_{1,\text{min}} := T_1(0) = f_1(k+2)(a\sigma_1(k+1) + (1-a)b_1\lambda_1(k)) \geq 0,
\]

\[
T_{2,\text{min}} := T_2(0) = f_2(k+2)(a\sigma_2(k+1) + (1-a)b_2\lambda_2(k)) \leq 0.
\]

At first, monotonic properties of \( R_{\lambda_i} \) (Eq. (11)) have been investigated. Therefore, the control law is rearranged:

\[
R_{\lambda_i}(u_i(k)) := -\frac{1}{1-b_i} \left[ b_i\lambda_i(k) + 1 - a \right] \frac{1}{a\sigma_i(k+1)} u_i(k) \geq 0,
\]

Because of \( f_1 > 0 \), \( R_{\lambda_1}(u_1) \) is strictly monotonically increasing (\( R_{\lambda_1}(u_1^1) < R_{\lambda_1}(u_1^2) \)), for \( u_1^1 < u_1^2 \) and \( R_{\lambda_2}(u_2) \) monotonically decreasing, since \( f_2 < 0 \).

Similar results are obtained for the functions \( T_1 \) (strictly monotonically increasing) and \( T_2 \) (strictly monotonically decreasing) with respect to the arguments \( r_{\lambda_1} \) and \( r_{\lambda_2} \) respectively.

\[ a) \] Case 1: \( R_{\lambda_1}(T_{1,d}) \geq 0 \wedge R_{\lambda_2}(T_{2,d}) < 0 \)

The calculated actuation variable \( r_{\lambda_2} \) would be negative and would be set to zero. Therefore, the requested torque for the second path \( i = 2 \) (always negative torques for this path) is bigger than the actually achievable torque \( T_{2,\text{min}} \):

\[
T_{2,\text{min}}(k+2) = T_1(0) = T_2(0) = T_2(0) < T_2(0) < T_2(0) = T_{2,d}(k).
\]

\[ b) \] Case 2: \( R_{\lambda_1}(T_{1,d}) < 0 \wedge R_{\lambda_2}(T_{2,d}) \geq 0 \)

The calculated actuation variable \( r_{\lambda_1} \) would be negative and would be set to zero. Therefore, the requested torque for the second path \( i = 2 \) is lower than the actually achievable torque \( T_{2,\text{min}} \):

\[
T_{2,\text{min}}(k+2) = T_1(0) = T_2(0) = T_2(0) < T_2(0) = T_{2,d}(k).
\]

\[ c) \] Case 3: \( R_{\lambda_1}(T_{1,d}) \geq 0 \wedge R_{\lambda_2}(T_{2,d}) \geq 0 \)

The calculated actuation variable \( r_{\lambda_1} \) would be negative and would be set to zero. Therefore, the requested torque for the second path \( i = 2 \) is lower than the actually achievable torque \( T_{2,\text{min}} \):

\[
T_{2,\text{min}}(k+2) = T_1(0) = T_2(0) = T_2(0) < T_2(0) = T_{2,d}(k).
\]
This is because $\mathcal{R}_2(T_{2,d}) < 0$ and $T_2$ is monotonically decreasing. With active constraints and without additional measures ($u_{a,i} = 0, i = 1, 2$) the resulting acceleration torque is smaller than the requested one $T_d$:

$$T_i(\mathcal{R}_{a,i}(T_{1,d}(k))) + T_{2,min}(k+2) < T_d(k), \quad (15)$$

By choosing the modified actuation variable $u_1 = T_{1,d} + u_{a,1}$ the requirement on the resulting torque can be achieved if

$$T_i(\mathcal{R}_{a,i}(T_{1,d}(k)) + u_{a,1}(k)) + T_{2,min}(k+2) = T_d(k) \quad (16)$$

is true.

Using (15) and (16) it can be shown that $u_{a,1} > 0$ and further $\mathcal{R}_{a,1}(T_{1,d} + u_{a,1}) > 0$ holds, because $T_{1,d} > 0$.

Now, eq. (16) is solved with respect to $u_{a,1}$:

$$u_{a,1}(k) = T_d(k) - T_{2,min}(k+2) - T_{1,d}(k). \quad (17)$$

The variables $u_i$ that fulfill the constraints for $r_{\lambda_i}$ and yield $T(k+2) = T_d(k)$ are then

$$u_1(k) = T_d(k) - T_{2,min}(k+2), \quad u_2(k) = T_{2,min}(k+2).$$

The corresponding activation variables are then

$$r_{\lambda_1}(k) = \mathcal{R}_{a,1}(T_d(k) - T_{2,min}(k+2)), \quad r_{\lambda_2}(k) = 0.$$

b) CASE 2: $\mathcal{R}_{a,2}(T_{1,d}) < 0 \land \mathcal{R}_{a,2}(T_{2,d}) \geq 0$

Because of symmetry reasons, the procedure is analogous to CASE 1. Here, the achieved torque for the muscle path 1 would be greater than the desired one (as the corresponding muscle cannot be deactivated fast enough). Therefore, the variable $u_2$ of the antagonistic pair has to be decreased by choosing:

$$u_{a,2}(k) = T_d(k) - T_{1,min}(k+2) - T_{2,d}(k). \quad (18)$$

The variables $u_i$ that fulfill the constraints for $r_{\lambda_i}$ and yield $T(k+2) = T_d(k)$ are then

$$u_1(k) = T_{1,min}(k+2), \quad u_2(k) = T_d(k) - T_{1,min}(k+2).$$

The corresponding activation variables are then

$$r_{\lambda_1}(k) = 0, \quad r_{\lambda_2}(k) = \mathcal{R}_{a,2}(T_d(k) - T_{1,min}(k+2)).$$

c) CASE 3: $\mathcal{R}_{a,1}(T_{1,d}) < 0 \land \mathcal{R}_{a,2}(T_{2,d}) < 0$

Because both constraints for the actuation variables are violated within CASE 4, the resulting accelerating torque would be $T_{1,min} + T_{2,min}$, which is not necessarily equal to the desired torque $T_d$. For determination of the compensation variables $u_{a,1}$ and $u_{a,2}$, two sub-cases are considered:

4.1) $T_d > T_{1,min} + T_{2,min}$ (actual torque too small)

4.2) $T_d < T_{1,min} + T_{2,min}$ (actual torque too big)

At first CASE 4.1) is evaluated. In order to achieve the desired accelerating torque $T_d$, the torque $T_1$ needs to be increased. This can be fulfilled by applying the additional desired torque $u_{1,a}$ such that

$$T_d(k) = T_i(\mathcal{R}_{a,i}(T_{1,d}(k) + u_{1,a}(k))) + T_{2,min}(k+2). \quad (19)$$

Because of (19) and the condition for CASE 4.1,

$$T_i(\mathcal{R}_{a,i}(T_{1,d}(k) + u_{1,a}(k))) \geq T_{1,min}(k+2) \geq 0$$

holds. Further, because $T_i$ is monotonically increasing, also $\mathcal{R}_{a,1}(T_{1,d} + u_{1,a}) \geq 0$ must hold, which means the constraint for the first actuation variable is satisfied. In this case, Eq. (19) becomes:

$$T_d(k) = T_{1,d}(k) + u_{1,a}(k) + T_{2,min}(k+2).$$

By rearranging, $u_{1,a}$ can be calculated leading to

$$u_{1,a}(k) = T_d(k) - T_{1,d}(k) - T_{2,min}(k+2).$$

The variables $u_i$ that fulfill the constraints for $r_{\lambda_i}$ and yield $T(k+2) = T_d(k)$ are then

$$u_1(k) = T_d(k) - T_{2,min}(k+2), \quad u_2(k) = T_{2,min}(k+2).$$

The corresponding activation variables are then

$$r_{\lambda_1}(k) = \mathcal{R}_{a,1}(T_d(k) - T_{2,min}(k+2)), \quad r_{\lambda_2}(k) = 0.$$

For CASE 4.2) an analog procedure leads to following result. The variables $u_i$ that fulfill the constraints for $r_{\lambda_i}$ and yield $T(k+2) = T_d(k)$ are then

$$u_1(k) = T_{1,min}(k+2), \quad u_2(k) = T_d(k) - T_{1,min}(k+2).$$

The corresponding activation variables are then

$$r_{\lambda_1}(k) = 0, \quad r_{\lambda_2}(k) = \mathcal{R}_{a,2}(T_d(k) - T_{1,min}(k+2)).$$

For all three cases it is straightforward to show that exact tracking of $T_d$ can only be achieved by inducing a co-contraction of the two muscles.

B. Two step ahead joint-angle prediction

The mechanical system (6) is time discretized:

$$x_1(k+1) = x_1(k) + T_a[x_2(k)] \quad (20)$$

$$x_2(k+1) = x_2(k) + \frac{T}{J}[-R_m x_2(k) - K_m x_1(k) + T(k)]. \quad (21)$$

By substituting $k \rightarrow k+1$ in Eq. (20), replacing $x_2(k+1)$ by Eq. (21) and further replacing $x_1(k+1)$ by the right hand side of Eq. (20) a two step ahead prediction for the joint angle is derived:

$$\dot{x}_1(k+2) = x_1(k) + T_a x_2(k) + T_a \left[ x_2(k) + \frac{T_a}{J}[-R_m x_2(k) - K_m x_1(k) + T(k)] \right]. \quad (22)$$

The accelerating torque $T(k)$ is replaced by the desired one $T_d(k-2)$, since $T(k)$ can not be measured and $T(k) = T_d(k-2)$ as enforced by the linearizing controller in the nominal case. The state variables $x_1$ and $x_2$, which refer to the joint angle $\theta$ and its derivation in time $\dot{\theta}$ are measured. Now, the estimates of $\dot{f}_i(k+2), i = 1, 2$, are determined by

$$\dot{f}_i(k+2) = k_{s,i}(\theta_{max,i} - \dot{\theta}(k+2)). \quad (23)$$
TABLE I

Table of Parameters

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V. LINEAR JOINT-ANGLE FEEDBACK CONTROL

On top of the linearizing controller, a linear, time-discrete and two degree of freedom joint angle controller is applied that uses the desired torque $T_d$ as its actuation variable. The mechanical model Eq. (6) is Euler discretized and used as transfer function for controller design:

$$ G(z) = \frac{T_d^2}{z^2 + (R_m T_d^2 - 2)z + (k_m T_d^2 - T_d R_m + 1)}/J. $$

The time delay of the system is neglected since it is considerable low compared to the rise time of $G$. The controller $K$ cancels the asymptotically stable plant $G$ and introduces a new second order transfer function to the open loop:

$$ K(z) = 1/G(z) \frac{a_0}{z^2 + b_1 z + b_0}. $$

The transfer function

$$ G_{cl}(z) = \frac{a_0}{z^2 + b_1 z + b_0 + a_0} \quad \text{(24)} $$

describes the reference to output behavior and is considered for the specification of the closed-loop behavior (assuming at first no pre-filter for the reference).

The parameters $a_0$, $b_0$ and $b_1$ are determined by a pole-placement procedure for the denominator of $G_{cl}$ and by claiming unity gain for $G_{cl}$ ($G_{cl}(z = 1) = 1$). The latter condition leads to an integral controller. A desired second order pole pair [1] described by a damping $d$ and a rise time $T_r$ is used for the pole-placement design.

Additionally to the feedback controller $K$, a reference pre-filter $V$ is designed to cancel the reference to output behavior $G_d$ and to introduce a slower second order dynamical system without conjugate complex poles:

$$ V(z) = 1/G_{cl}(z) \frac{(1 - \exp(-s_1 T_d) \cdot (1 - \exp(-s_2 T_d))}{(z - \exp(-s_1 T_d) \cdot (z - \exp(-s_2 T_d)). $$

VI. RESULTS

For demonstrating the effectiveness of the linearizing controller along with its extension for the given constraints, a comparison of the controller with and without the extension was performed in form of a simulation study. Predefined trajectories were fed to the desired torques $T_d$ and $T_{d,cc}$, while the torques of the plant model (cf. sec. III.) were evaluated. For the model and the linearizing controller the parameters described in Tab. I(a) were used. In the Figs. 6 and 5 the results in form of a comparison of the desired torques $(T_d, T_{d,cc})$ and the actual torques $(T, T_{cc})$ are presented. Additionally, the actuation variables are shown.

![Fig. 5 - Results for the linearization controller without taking constraints on the actuation variables into account. The actuation variables shown in the lower subplot also show the computed negative values.](image)

![Fig. 6 - Results for the extended linearization controller.](image)

For the linearizing controller without the extension, there is a considerable mismatch of $T$ compared to the reference...
The feasibility of the developed approach is further demonstrated by a joint-angle control experiment for one healthy subject. Before the control experiment, the $\lambda$-Control loops were tuned and the model (cf. sec. III.) was identified using the procedure described in [11]. The resulting parameters are summarized in Tab. I(b). The results for the control experiment for a stepwise changing reference are shown in Fig. 7. The desired co-contraction torque $T_{cc,d}$ was zero since also no co-contraction was present in the experiment for parameter identification. The resulting co-contraction torque was of course temporally greater than zero to fulfill $T = T_d$ when violating actuation variable constraints. Additionally to the measured joint angle $\vartheta$, the nominal behavior $\vartheta_{nom}$ (if the joint angle controller is applied to the linear plant model $G$) is shown. As the step size increases, the overshoot of the joint angle increases as under an increasing distance of $\vartheta$ to its equilibrium, the validity of the assumed model decreases.

Using two underlying $\lambda$-Controllers for each muscle respectively, the inner control system rapidly compensates for fatigue and changing muscular thresholds, which is a prerequisite for the practical application of neuro-prosthetic systems in clinical environments.

Further investigation of the influence of co-contractions on the mechanical stiffness and elasticity is ongoing. In this field, the authors expect a great potential for the artificial control of limb movements.

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