Exercise 1

Exercise 1.1 (Dental Clinic)
At a dental clinic, a dentist and two dental nurses work together. When patients arrive, they wait in the waiting room before being treated. Two kinds of treatment are performed: tooth cleaning and medical examination. Dental tooth cleaning is done by one of the nurses. A medical examination needs to be done by the dentist and a nurse. When a treatment is finished, the patient leaves the clinic.

a) Use a Petri net to model the dental clinic.

b) State matrices $A^{-}$ and $A^{+}$, and obtain the incidence matrix $A$. Suppose three patients have arrived and two of them are having their teeth cleaned. Use the matrices to determine whether at this situation it is possible for a medical examination to start.

c) The dental clinic is a small place, therefore a maximum of three people can be in the waiting room at the same time. What are the changes that need to be introduced in the Petri net from a) to take this fact into account?

d) Suppose now that the clinic will start offering dental surgery, which requires the participation of the dentist and both nurses. Change the previous model so as to incorporate this new kind of treatment.

Exercise 1.2 (Workshop)
At a workshop two operations can be done: painting and polishing. Both operations need to be performed in the specified order for each component: first painting, then polishing. Before processing, the components are kept in a depot with a capacity of 5 items at a time. Due to a lack of tools, it is only possible to paint two and to polish three components at the same time.

a) Use a Petri net to model the workshop.

b) State the incidence matrix.

c) Provide the corresponding reachability graph for the case where two components are kept in the depot and no new components arrive.

Exercise 1.3 (Street Narrowing)
A traffic light system is used to control the traffic at a single-line street segment (Figure 1). Cars can arrive from the left and from the right. In order to prevent collisions, the system is programmed to avoid that there are two cars in the single-line segment (i.e., between the two traffic lights) at the same time.

a) Use a Petri net to model the system.

b) State the incidence matrix.
c) Provide the corresponding reachability graph for the case where two cars wait on each side of the single line segment. For simplicity, assume that no new cars are going to arrive from either side.

d) Assuming that there are many cars waiting on both sides of the traffic light, modify the Petri net model from a) such that the cars can pass the narrowing alternately from the left and right side.

e) Does the resulting Petri net from d) have a specific structure? (Hint: see Section 2.3 of the Lecture Notes)