Modeling and control of prosumer-based microgrids: a Petri net approach*

Lia Strenge, Germano Schafaschek, and Jörg Raisch

Abstract—The transformation towards renewable energy sources is a global phenomenon. While industrialized regions are transitioning from large fossil-fueled power plants towards increasingly decentralized renewable energy sources, so far unelectrified regions in resource-constrained communities are replacing kerosene lamps and candles by household-based stand-alone solar home systems (SHSs). The latter can form a bottom-up power grid when interconnected. Both scenarios may lead to prosumer-based microgrids. Each grid node is a prosumer, hence it can sell and buy energy to and from the other prosumers. In this paper, we derive a Petri net model for the interconnection of an arbitrary number of prosumers forming a prosumer-based microgrid. The proposed control strategy addresses fair and efficient energy sharing. These control objectives are formalized as a local and a global control problem, respectively. A systematic method to implement them via supervisory control in a least restrictive way is presented, thereby enforcing fair and efficient microgrid operation.

I. INTRODUCTION

The global energy transformation towards renewable energies and the need for electrification has recently led to new power grid structures. Microgrids have arisen as a concept for local energy balancing in grid-connected or stand-alone modes. In an industrialized context, they can be part of an active distribution grid, working independently in case of failure. On islands and in resource-constrained communities, they may be the only source of electricity available [14], [33]. An even more recent concept is peer-to-peer energy trading among prosumers in a microgrid [35]. Each prosumer can produce/generate and consume energy locally and sometimes also store energy. In addition, energy can be fed into a power grid (feeding in) and be drawn from a power grid (consuming) or, in some cases, not shared (stand-alone/idle). To schedule the energy sharing, different energy management strategies exist: in [21], cooperative operation is proposed while in [22] game theory is applied; in [25], the authors compare a variety of centralized and distributed energy management concepts; in addition, market-based approaches [6], [18], [26] vs. energy commons and cooperatives [11], [12] have been discussed. With a focus on scaling reduction by substructures, propositions include a fractal structure [8], virtual power plants of peer-to-peer transactions [24], prosumer community groups [29], and hierarchical approaches [10], [36].

From a control-oriented point of view, existing work assumes fixed feed-in and consuming nodes [2] or uses heuristic control methods with focus on implementation and communication: in [32], decentralized control based on flow charts for power deals and state-of-charge based strategies are compared; in [30], consensus-based controllers with local optimization for directed communication graphs are designed; in [1], allocation algorithms with asymptotic optimality for internet-of-things enabled prosumer markets with high uncertainty are presented.

Formal and systematic approaches to control the switching of prosumers between feeding in and consuming have not been extensively exploited in microgrids. In this paper, we propose such an approach using Petri nets as the base formalism. Petri nets are a well established tool to model and control discrete-event systems, i.e., systems with a discrete state space where state transitions are caused by the abrupt occurrence of events at discrete (and, in general, irregular) time instants [3], [23]. Petri-net-based distributed supervisory control has recently been applied to automated manufacturing systems [15], [16]. Concerning power systems, Petri nets mostly appear in modeling of security-related events [5], [9], [20], [28] and of the energy management for microgrids with fixed feed-in and consuming nodes [4], [7] or for a single grid-connected prosumer [19]. Storage management is addressed by state-of-charge related Petri net models in [27], [31]. Colored Petri nets are used in [34] for performance analysis and in [17] for fault recovery of smart grids.

The contribution of this work is the systematic application of local and global supervisory control based on a Petri net model to a network of prosumers, i.e., a peer-to-peer microgrid. Social and ecological objectives are considered for the control. The underlying application is a microgrid of formerly islanded solar home systems. The results, however, apply to any prosumer-based microgrid.

The paper is structured as follows. In Section II, the preliminaries regarding prosumer-based microgrids, Petri nets, and supervisory control with Petri nets are introduced: in Section III, Petri net models for 1 and \( N > 1 \) prosumers are derived; Section IV covers the local and global control synthesis and the overall closed-loop model, and Section V is the conclusion.

II. PRELIMINARIES

In this section, we introduce the concept of a prosumer-based microgrid (Section II-A); we also recall some fundamentals of Petri net theory [3] and Petri net control [23] (Section II-B).
A. Prosumer-based microgrid

A solar home system (SHS) in a household is composed of a generation unit (here a solar panel), a storage unit (here a battery), and loads (e.g., lights, fan and TV). Therefore, a SHS can work completely stand-alone within its limitations. In order to improve the energy service provision, SHSs, e.g., in a village, can be interconnected by additional power lines to form a power grid from the bottom up. Like this, energy can be shared among the households. The first installed so-called swarm grid in rural Bangladesh covered around 8 households [13]. This setting is a special case of a prosumer-based microgrid and the inspiration for this work. A prosumer-based microgrid is a power network with arbitrary topology and size where each grid node can be feeding in, consuming, or idle with respect to the power grid. Hereafter, grid nodes are synonymously denoted as users or prosumers, and exemplarily as SHSs. Power grid is abbreviated by grid. Please note that in this work the interaction of prosumers with the grid is studied. It does not cover details on the internal energy management between local generation, storage and loads of the individual users.

B. Petri nets basics

A Petri net is a tuple \((P, T, A^+, A^-, E, x^0)\), where \(P\) and \(T\) are the (finite) sets of places and transitions, respectively, \(A^+ \subseteq P \times T\) and \(A^- \subseteq T \times P\) are weight matrices, \(E = \{(p_i, t_j) | A^-_{ij} \geq 1\} \cup \{(t_j, p_i) | A^+_{ij} \geq 1\}\) is the set of directed arcs, and \(x^0 \in \mathbb{N}_0^n\) is the vector of initial markings.

In graphical representations, places are shown as circles, transitions as bars, and arcs as arrows (see Fig. 1). The initial marking is shown by drawing, for each place \(p_i \in P\), \(x^0_i\) dots (or tokens) within the circle representing \(p_i\).

A Petri net can be seen as a discrete-event dynamical system whose events are represented by the transitions and whose state corresponds to the distribution of tokens among the places. Define the state signal \(x : \mathbb{N}_0 \rightarrow \mathbb{N}_0^n\), where \(x_i(k)\) represents the number of tokens in place \(p_i\) when \(k\) (not necessarily distinct) transitions have occurred; the initial state is \(x(0) = x^0\). Define, also, the incidence matrix \(A = A^+ - A^- \in \mathbb{Z}^{n \times m}\). The system’s dynamics can then be described by two rules: (1) a transition \(t_j \in T\) can occur (or fire) in state \(x(k)\) if and only if \(\forall p_i \in P, x_i(k) \geq A^-_{ij}\); (2) if \(t_j\) fires in state \(x(k)\), the resulting new state, \(x(k+1)\), is given by \(x_i(k+1) = x_i(k) + A_{ij}, i \in \{1, \ldots, n\}\).

C. Supervisory control with Petri nets

It is possible to restrict the evolution of a Petri net’s state by control action so as to respect a given set of linear inequalities\(^1\) \(\gamma_i x(k) \leq b_i\), \(\gamma_i \in \mathbb{Z}_{\geq 0}^n, b_i \in \mathbb{Z}^n, i \in \{1, \ldots, q\}\). These can be written as \(\Gamma x(k) \leq b\), where \(\Gamma \in \mathbb{Z}^{q \times n}\) and \(b \in \mathbb{Z}^q\). The control mechanism is to prevent the firing of certain transitions; this is attained by adding \(q\) controller places \(p_{c1}, \ldots, p_{cq}\) to the original Petri net.\(^2\)

\(^1\)Here and throughout the paper, \(|\cdot|\) denotes the cardinality of a set and \(M_{ij}\) is the element in row \(i\) and column \(j\) of matrix \(M\).

\(^2\)Such inequalities are always to be understood as being valid for all \(k \in \mathbb{N}_0\).

The initial marking in the controller places can be obtained by \(x^0_c = b - \Gamma x^0\), and their connection with the system’s transitions by the incidence matrix \(A_{c} = -\Gamma A \in \mathbb{Z}^{q \times m}\); there is an arc from \(t_j\) to \(p_{c}\) (with weight \((A_{c})_{ij}\) if and only if \((A_{c})_{ij} > 0\), and there is an arc from \(p_{c}\) to \(t_j\) (with weight \(-(A_{c})_{ij}\)) if and only if \((A_{c})_{ij} < 0\). The resulting closed-loop Petri net has state \(x(k+1)\) and incidence matrix \(A_{c}\), and it operates according to the same rules introduced in Section II-B. A transition \(t_j \in T\) is affected by the controller if and only if \((A_{c})_{ij} < 0\) for some \(i \in \{1, \ldots, q\}\). The controller synthesized through the procedure outlined above is least restrictive, in the sense that a transition firing is prohibited if and only if it would lead to the violation of the specification. In many systems, not all events are subject to disenablement, which, in the Petri net context, translates into some transitions being uncontrollable.\(^3\) A controller is implementable if it does not try to prohibit the firing of any uncontrollable transition, i.e., if \(\forall t_j \in T, t_j\) uncontrollable \(\Rightarrow [\forall i \in \{1, \ldots, q\}, (A_{c})_{ij} \geq 0]\).

III. MODELING

In this section, we derive a Petri net model for one user (Section III-A) and then for a prosumer-based microgrid of \(N \in \mathbb{N}\) users (Section III-B).

A. Petri net model for one prosumer

There are three main aspects of a prosumer that we would like to capture in our Petri net model. A first crucial feature is the user’s sharing status towards the power grid; as pointed out in Section II-A, from the point of view of the grid a prosumer can be feeding in, consuming, or idle. However, a user cannot decide on its own to actually start feeding in, as this obviously depends on some other users being willing to consume; analogously, for a user to start consuming there have to be other users capable of feeding in energy. What each user can do individually is to be ready to feed in (R2F), idle (ID), or ready to consume (R2C); whether a user which is R2F/R2C is actually feeding in/consuming will then be determined by the sharing roles of the other users, as shall be further explained in Section III-B. The sharing role is clearly a discrete variable by nature, and the mechanism through which a user can switch between roles can be directly represented by means of three places and four transitions, as shown in the middle part of Fig. 1.

A second fundamental aspect our model should reflect is the overall net amount of energy that has been shared by a user. As opposed to the sharing role, this is an intrinsically continuous variable; it is, nonetheless, quite natural to divide the whole range of values into a discrete set of intervals, the ones adopted here being high energy feed-in (HEF), high energy consumption (HEC), and balanced energy sharing (BES). Each pair of neighboring intervals is divided by a threshold, whose nominal value can be arbitrarily chosen (provided consistency is assured); the actual value can then
vary around the nominal one (for instance, via control action) during operation. The crossing of such a threshold is an event (transition firing) in our system; the energy-sharing aspect is modeled by the three places and four transitions on the right part of Fig. 1. As can be seen in the model, when transition $t_{E1}$ fires, the user is cut off from feeding in (consuming) and forced to become idle; this, combined with control action (see Section IV-B), aims at preventing feed-in monopolies and unbridled energy consumption.

The third and last element to be modeled is the state of charge of the user’s energy storage unit (e.g., battery). This is again a continuous quantity, but once more thresholds can be defined which divide the whole range of values (0 to 100%) into three sections: low (SL), medium (SM), and high (SH), as represented by the three places and six transitions on the left part of Fig. 1. A clarification is in order: the event of crossing the threshold from SM to SH can occur under two fundamentally different circumstances, hence it being represented by two transitions ($t_{S2}$ and $t_{S3}$). The firing of $t_{S2}$ corresponds to the state of charge of the storage becoming high while the user is idle towards the grid, meaning it is locally charging its storage. Transition $t_{S3}$, on the other hand, fires when the user is consuming energy from the grid to charge its storage, in which case a mechanism is assumed to exist that cuts the user off from consuming and forces it to switch to idle as soon as the state of charge of the storage becomes high. This assures that self-sufficiency by local prosumption is incentivized. In a grid with energy scarcity, it additionally prevents users from storing energy for economic reasons while other users might need it more urgently. Note that the user is not allowed to immediately become R2C again after being cut off, as $t_{R3}$ can only fire if the state of charge of the storage is low. An analogous reasoning can be applied to $t_{S5}$ and $t_{S6}$, the mechanism then enforcing that a user does not continue to feed energy into the grid when the state of charge of its own storage is low. This also encourages self-sufficiency rather than seeking short-term economic benefit; in addition, it prevents a battery from being too deeply discharged, thus improving life-time.

The nomenclature and meaning of the places in the single-user model is summarized in Table I. By observing the directions of the arcs in Fig. 1, the meaning of the transitions can easily be extracted. The state is given by the distribution of tokens among places $p_1$ to $p_9$. The presence of a token in places $p_1$, $p_9$, and $p_8$, for instance, represents that the user’s current sharing role is ready to feed in, idle, and ready to consume, respectively. It should be clear that, at any point during the evolution of the system, exactly one of these three places will contain a single token, and the other two will be empty; the current sharing role is thus always uniquely determined. An analogous reasoning applies to places $p_1$, $p_2$, and $p_3$ regarding the state of charge of the storage unit, as well as to places $p_7$, $p_8$, and $p_9$ with respect to the energy-sharing status. The initial state can, in principle, be arbitrarily chosen among all coherent states (in the sense of the above discussion); for simplicity, we assume throughout the paper that the initial state is as shown in Fig. 1, i.e., the user’s initial sharing role is idle, the initial state of charge of the storage is medium, and the net shared energy is also balanced.

Hereafter, let $n$ be the number of places and $m$ the number of transitions of the single-user model. One can see from Fig. 1 that $n = 9$ and $m = 14$. The incidence matrix $A \in \mathbb{Z}^{n \times m}$ has the form

$$
A = \begin{bmatrix}
A_{SS} & A_{SR} & A_{SE} \\
A_{RS} & A_{RR} & A_{RE} \\
A_{ES} & A_{ER} & A_{EE}
\end{bmatrix},
$$

where $S$, $R$, and $E$ stand respectively for storage, role, and energy, and the block $A_{XY}$ represents the relation between
the places from $X$ and the transitions from $Y$, with $X, Y \in \{S, R, E\}$. The values of all entries are shown below:

$$A = \begin{bmatrix}
-1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}. \quad (1)$$

As mentioned in Section III-A, typically some transitions in a Petri net are controllable (meaning the corresponding events in the underlying system are subject to disablement) whereas others are not. For the model described above, it is reasonable to designate transitions $t_{R1}$ and $t_{R3}$ as controllable, as they represent user decisions which may be impeded from occurring (becoming ready to feed in or to consume); for example, such decisions can simply be overruled by physically disconnecting a user from the grid. Users should always be free, however, to choose to cease being ready to feed in or consume, meaning that transitions $t_{R2}$ and $t_{R4}$ are considered to be uncontrollable. Moreover, according to the foregoing discussion, the firing of energy- and storage-related transitions ($t_{E1}$ to $t_{E4}$ and $t_{S1}$ to $t_{S6}$, respectively) represents the crossing of the corresponding thresholds; inasmuch as the values of such thresholds can be varied during operation, all these transitions can be considered to be controllable.

### B. Overall model for $N$ users

We now consider a prosumer-based microgrid with $N \in \mathbb{N}$ users, e.g., consisting of $N$ interconnected SHSs, each of which is modeled by a Petri net like the one in Fig. 1. Notice that the nominal storage and energy thresholds and their variability ranges need not be the same for all users, and their choice has no impact on the structure of the model. The nominal energy threshold, in particular, can be adapted for each user according to the type of usage, so that a user with a high load due to business, e.g., an internet café, might have a high consumption margin, whereas one with large generation facilities could be granted a larger feed-in margin.

Henceforth, we denote with a superscript $(i)$ the places and transitions corresponding to user $i \in \{1, \ldots, N\}$, so that, e.g., $p_{(i)}^{(0)} = \text{ID}^{(i)}$ refers to the place representing the idle role of user $i$. Let $A^{(i)}$ be the incidence matrix of user $i$. We have, for all $i \in \{1, \ldots, N\}$, that $A^{(i)} = A_i$ with $A_i$ as in (1). The incidence matrix $A_{N\text{-user}}$ can then be constructed:

$$A_{N\text{-user}} = \begin{bmatrix}
A^{(1)} & 0 & \ldots & 0 & 0 \\
0 & A^{(2)} & 0 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \ldots & A^{(N-1)} & 0 \\
0 & 0 & \ldots & 0 & A^{(N)} \\
\end{bmatrix}. \quad (2)$$

$A_{N\text{-user}}$ has dimensions $nN \times nN = 9N \times 14N$. It is clear that the number of places and transitions in the model grows linearly with the number of users.

Up to this point, the model includes no relation among the places and transitions of different individual prosumers, as evidenced by (2). We adopt the convention that user $i$ is feeding into the grid if there is a token in place $R2F^{(i)}$ and in place $R2C^{(j)}$ for some $j \neq i$, i.e., if user $i$ is ready to feed in and there is at least one other user ready to consume (which may or not already be consuming). Similarly, user $i$ is consuming if it is ready to consume and there is at least one other user ready to feed in. Thus, even though there is no explicit interconnection among the users in the model, the distribution of tokens in the role-places tells us whether there is energy being shared in the grid. More relevant relations among users will be enforced by control (Section IV-B).

### IV. CONTROL

In this section, we first state and solve a local control problem for each prosumer addressing fair sharing (Section IV-A) and then a global control problem for the overall grid to avoid energy waste (Section IV-B).

#### A. Local control

As mentioned in Section III-A, the model of Fig. 1 assumes a mechanism which automatically cuts off from feeding in/consuming a user that crosses its energy threshold to HEP/HEC. However, as long as its state of charge of the storage allows it, there is nothing preventing the user from switching right back to R2F/R2C after being cut off.

With this in mind, we define a local specification for each user $i \in \{1, \ldots, N\}$ as follows: as long as the allowed consumption margin of a user is exceeded, the user is not allowed to further draw from the grid. In terms of the Petri net from Fig. 1, this translates into the restriction that the presence of a token in places $R2C^{(i)}$ and $HEC^{(i)}$ must be mutually exclusive, and the total amount of tokens in these two places combined must not exceed 1. Analogously, we state that as long as the allowed feed-in margin of a user is exceeded, the user is not allowed to further feed into the grid, meaning that the presence of a token in places $R2F^{(i)}$ and $HEF^{(i)}$ must be mutually exclusive. Defining the state vector $x^{(i)} \in \mathcal{N}^n$ to represent the marking in the places of user $i$ (i.e., $x^{(i)}_j$ refers to place $p^{(i)}_j$, $j \in \{1, \ldots, n\}$; see Table I), these specifications can then be expressed as

$$x^{(i)}_6(k) + x^{(i)}_9(k) \leq 1,$$

$$x^{(i)}_4(k) + x^{(i)}_7(k) \leq 1,$$

or in the form $\Gamma^{(i)} x^{(i)}(k) \leq b^{(i)}$ as in Section II-C:

$$\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} x^{(i)}(k) \leq \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \quad (3)$$

By applying the synthesis procedure discussed in Section II-C, we obtain the local supervisors with initial marking $x^{(i)}_c = \begin{bmatrix} 1 \end{bmatrix}$ and incidence matrix

$$A^{(i)}_c = -\Gamma^{(i)} A^{(i)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}. \quad (4)$$
As well as in the following discussion, the superscript \(^{(i)}\) in the names of places and transitions is omitted for readability.

Fig. 2 shows the closed-loop model for one user, with the two local controller places named LC1 and LC2. In the figure as well as in the following discussion, the superscript \(^{(i)}\) is omitted for readability. One can observe that the transitions affected by the local supervisor are \(t_{R1}\) and \(t_{R2}\). For instance, suppose a user switches from ID to R2C, so that LC1 becomes empty. If the user then chooses to stop consuming \((t_{R4})\) or gets cut off because its state of charge of the storage has become high \(t_{S3}\), a token is added to LC1 so that the controller does not interfere with the switching between roles. If, however, the user is R2C and gets cut off due to crossing the threshold from BES to HEC \((t_{E3})\), no token is returned to LC1, so the switching from ID to R2C (i.e., the firing of \(t_{R3}\)) is disabled. LC1 will receive a token again when the shared energy balance is restored, meaning the threshold from HEC to BES is crossed \((t_{E4})\); only then is the user allowed to switch its role to R2C. An entirely analogous reasoning can be applied to investigate the effect of LC2 in disabling \(t_{R1}\). It is clear that the transitions are only disabled when their firing would cause the violation of the specification \((3)\). Also worth observing is the fact that the local controllers are implementable (see Section II-C), as they do not try to disable any uncontrollable transition.

### B. Global control

Besides addressing fair sharing among the users, we aim at reducing the excess energy in the grid. Excess energy is energy that is available but cannot be used, e.g., in a stand-alone SHS when the sun is shining and the battery is full. Sharing such energy is one of the main motivations for interconnecting previously islanded SHSs to form a prosumer-based microgrid. Considering a grid with \(N\) users modeled as in Section III, and assuming each user is in a closed loop with its local controller (Section IV-A), excess energy can arise if users that are R2F and users that are R2C are simultaneously cut off. Therefore, we design a global controller to enforce the following rule: if at least one user is cut off due to high energy consumption, no user should be cut off due to high energy feed-in. This means that, if HEC\(^{(i)}\) has a token for some \(i \in \{1, \ldots, N\}\), then HEC\(^{(j)}\) must have no tokens for all \(j \in \{1, \ldots, N\}\); in other words, for any \(i, j \in \{1, \ldots, N\}\), the presence of tokens in places HEC\(^{(i)}\) and HEC\(^{(j)}\) must be mutually exclusive. Recalling that \(x_2^{(i)}\) and \(x_5^{(i)}\) represent the marking in places \(p_2^{(i)} = \text{HEF}^{(i)}\) and \(p_5^{(i)} = \text{HEC}^{(i)}\), respectively, and that \(x_2^{(i)}(k)\) and \(x_5^{(i)}(k)\) can only assume values in \([0, 1]\) for any \(k \in \mathbb{N}_0\), this rule can be formally stated as

\[
x_2^{(i)}(k) + x_5^{(j)}(k) \leq 1, \quad \forall i, j \in \{1, \ldots, N\}, \quad i \neq j.
\]  

(5)

Note that we can exclude \(i = j\) because no prosumer can have high energy consumption and feed-in at the same time.

We also want to enforce the converse rule, namely that if at least one user is cut off due to high energy feed-in, no user should be cut off due to high energy consumption; i.e., if HEC\(^{(i)}\) has a token for some \(i \in \{1, \ldots, N\}\), then HEC\(^{(j)}\) must have no tokens for all \(j \in \{1, \ldots, N\}\). Therefore, this again amounts to the presence of a token in places HEC\(^{(i)}\) and HEC\(^{(j)}\) being mutually exclusive for any \(i, j \in \{1, \ldots, N\}\), which leads to the same set of inequalities as in (5).

Let \(\hat{x} = [x_1, x_2, \ldots, x_4, x_{11}, x_{12}, x_{13}, x_{14}, x_{21}, x_{22}, x_{23}, x_{24}] \in \mathbb{N}_0^{(n+2)}\) denote the state vector of user \(i\) in closed loop with its local controller, where \(x_1, x_2, x_3, x_4\) correspond to the local controller places LC1, LC2, LC3, and LC4, respectively, and \(\nu \) denotes the transpose of a vector \(\nu\). Now, define the global state vector \(\hat{x}_{\text{glob}} = [\hat{x}^{(1)}, \hat{x}^{(2)}, \ldots, \hat{x}^{(N)}] \in \mathbb{N}_0^{(n+2)N}\). The \(N(N-1)\) inequalities in (5) can be directly expressed in the form
by the latter in the energy thresholds; only when a threshold
Supervisors are not directly affected by the global supervisor,
size of practical prosumer-based grids. Larger power grids
2 loop one including local and global controllers, a total of
Variable energy thresholds reduce predictability and direct
For user \(i\) in closed loop with its local controller, and let
Let \(A^{(i)} = \begin{bmatrix} A^{(i)}_{c,N} & A^{(i)}_{c,N} \end{bmatrix} \in \mathbb{Z}^{(n+2) \times m}\) be the incidence matrix
for each \(i \in \{1, \ldots, N\}\). Applying the synthesis procedure
from Section II-C, we get \(x^{0}_{N,usr} = b_{N,usr}\) and \(A_{c,N,usr} = -N_{usr} A_{N,usr}\). Considering again the example with
For user \(i\) affected by the global supervisor
is cut off due to high energy consumption (HEC\(^{(j)}\)), another user (say, user \(\ell\)) may exceed its nominal feed-in threshold without being
cut off; in order to allow supplying remaining consumers,
the threshold is raised by the global supervisor and is not
effectively crossed. As pointed out in Section IV-B, the feed-
in threshold of user \(\ell\) is restored to its nominal value as soon as
user \(j\) switches from HEC\(^{(j)}\) back to BES\(^{(j)}\) (assuming no other user has been cut off for high consumption); in
this case happens while user \(\ell\) still has a feed-in level
higher than its nominal feed-in threshold, it is then implicitly
assumed that user \(\ell\) will be cut off, switching from BES\(^{(j)}\) to HEF\(^{(j)}\). This mechanism, however, is not captured in our
closed-loop model, since the firing of transition \(t_{E_{1}}^{(j)}\) in
the described situation is merely possible, but not guaranteed.
This limitation comes partly from the “permissive” nature of the
supervisory control framework we adopt, seeing as the
only action a supervisor can take is to enable/disable events,
but it has no means to force or prioritize the occurrence of a
particular enabled transition.
Another tradeoff between transparency and possible abuse
may arise in the threshold management itself. Nevertheless,
one advantage of the control is that a specific user cannot easily
abuse the energy sharing for its own economic profit. The
variable energy thresholds reduce predictability and direct
the focus to the local balance of feed-in and consumption,
so that the grid stays/becomes an add-on instead of turning
into a mere utility. In addition, once a consumer is cut off
as described above, other users that cross their consumption
thresholds are also cut off normally. So, the users that are
feeding in can only continue to do so above their nominal
limits until the point when all the consumers are cut off and
there is none left. Again, an analogous reasoning follows for
the case of cutting off users that are feeding in. Thus, there
is no room for unlimited feed-in or consumption.
5. CONCLUSION
We presented a Petri net model for a prosumer-based
microgrid and proposed local controllers to enforce fairness
in the energy sharing and a global controller to enforce
excess-energy reduction. The overall closed-loop Petri net for
4 interconnected prosumers (grid nodes) comprises \(N(N + 10)\) places (9\(N\) from the open-loop model plus 8\(N\) from the controllers) and 14\(N\) transitions, so that the total size (places + transitions) of the model scales with \(N^2\). We emphasize that, starting solely from the single-user model and an arbitrary number (\(N\)) of users, our approach allows to automatically obtain the global open-loop model, the local and global control specifications, as well as the overall closed-loop model respecting the specified behavior with minimal restriction (maximal permissiveness). Hence, the
main contribution of this work with respect to existing literature is a formal and systematic method to employ supervisory control theory with Petri nets to the increasingly important application of prosumer-based microgrids. The drawbacks of the approach (discussed in Section IV-C) may be mitigated by the use of more sophisticated Petri net models, e.g., with inhibitor arcs. More detailed research is also needed on the definition and practical implementation of (dynamic) discretization thresholds. For a complete operational model, the integration of continuous dynamics for monitoring and controlling the energy sharing may need to be explored. For larger grids, a division into subgrids seems beneficial but would require a suitable subgrid interconnection model and possibly an additional control layer. Using colored Petri nets is another promising avenue which may aid in tackling large grids, especially since we consider the same basic model for each individual user.

ACKNOWLEDGMENT

Thanks to ME SOLshare Ltd. for the insights, Wening Wu and Georges Francky Njie for their collaboration, and Tobias Bengfort for the discussions.

REFERENCES


