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Output Tracking of a Bioreactor with Nonminimum Phase Characteristic

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Abstract

A framework for the output tracking of nonlinear nonminimum phase systems is presented. The control problem is solved by a two degree of freedom design. A feedforward part generates an explicit trajectory for the nominal input and states by an analytic inversion of the plant to let the plant output follow an externally given reference trajectory. A gain scheduling controller stabilises the plant around these trajectories, the parameters of which are scheduled in dependence on the current nominal control signal and the nominal state vector. The design of the controller is based on a linear parameter varying model, which results from the linearisation of the plant about the nominal trajectories. An overview of techniques for the stable inversion of nonminimum phase systems is given. The approach is demonstrated on a bioreactor which shows maximum phase behaviour.

1 Introduction

The tracking control of nonlinear systems represents a significant problem in current research in control engineering. At the end of the eighties, using geometric methods, new nonlinear approaches were developed, for example input-output linearisation [1]. A major drawback of this method is that by application to nonminimum phase plants the closed-loop becomes internally unstable. Nonminimum phase characteristics is a concept which is only related to the input-output behaviour of a system. The "inverse response behaviour" represents the significant effect of such systems. As is well-known from linear control theory nonminimum phase systems result in a limitation of the achievable closed-loop performance. Because of the unsuitability of many nonlinear control approaches, in general controllers based on linear methods, like gain-scheduling, will be applied. This leads inevitably to reduced performance. In this paper we present a two degree of freedom (2DOF) structure, combining the tracking performance of nonlinear control approaches, like input-output linearisation, with the robustness of linear approaches, like gain-scheduling [2]. It will also be shown how the problem of inversion of nonminimum phase systems can be approximately solved.

2 Two Degree of Freedom Structure

This paper deals with the two degree of freedom structure (2DOF) shown in fig. 1. Similar structures are proposed in

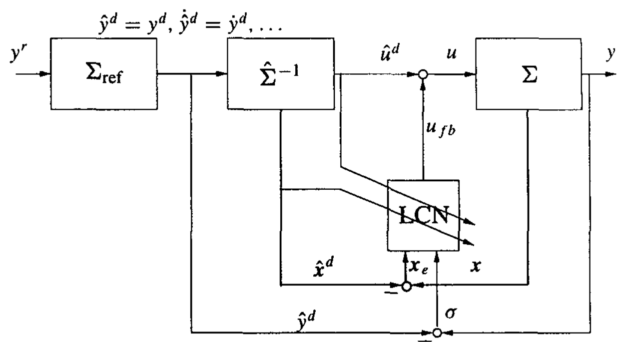

Figure 1: 2DOF structure consisting of an (approx.) inverse $\hat{\Sigma}^{-1}$ of the plant model Σ as feedforward controller and a local controller network (LCN) in the feedback loop

[3, 4], however, including different components. In our structure an analytic inverse Σ^{-1} or its approximation $\hat{\Sigma}^{-1}$ of the plant Σ is contained as a feedforward controller. From the inverse plant a nominal control action u^d and a nominal state trajectory x^d are determined to let the plant output y follow a given smooth reference signal y^d. Sufficiently smooth signals can be generated by filtering an external reference signal y^r with the system Σ_{ref} which has at the same time a determining influence on the tracking performance. Due to the assumption of an unknown future trajectory of the reference signal y^d the calculation of the required signals has to be done in real time. In the case of nonminimum phase systems an exact causal solution does not exist in real time so that a minimum phase approximation $\hat{\Sigma}$ of the original system has to be used for the determination of approximated variables $\hat{u}^d$ and $\hat{x}^d$ for the required nominal trajectories. In order to reduce the output tracking error $\sigma = y - y^d$ resulting from using an approximated inverse, external disturbances, an unstable open-loop plant, system parameter changes or different initial states of plant and inverse, an underlying stabilising feedback controller is required.

A network of local controllers [5] has been investigated as a possible realisation. For the local linear controllers we choose simple state feedback designed using a linearisation of the nonlinear plant along the tracking trajectories. To achieve a vanishing output tracking error for constant reference signals integral action was introduced into the controller. Consequently, the additional control action u_{fb} is calculated using the tracking error x_e of the states and the output error as input variables of the local controller network (LCN), the parameters of which are scheduled in dependence on the current values of $\hat{u}^d$ and $\hat{x}^d$.

3 System Inversion

We consider SISO nonlinear dynamical systems which are affine in the control variable

$$\Sigma: \quad \begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x), \qquad x \in \mathbb{R}^n. \end{aligned} \tag{1}$$

Here $f(x)$ and $g(x)$ are vector functions and $h(x)$ is a scalar function. The inversion problem can be described by

$$\begin{aligned} \dot{x}^d &= f(x^d) + g(x^d)u^d \\ y^d &= h(x^d). \end{aligned} \tag{2}$$

The nominal state vector x^d and nominal input u^d have to be determined for a given output signal y^d in order to fulfil (2).

3.1 Minimum phase systems

To enable the solution of the above mentioned inversion problem (2) a form of (1) is required which provides a more usable relation between the output and control variables. By performing a transformation of (1) into Byrnes-Isidori normal form using a local diffeomorphism $(\zeta^T, \eta^T)^T = \Phi(x)$ a suitable description is given as

$$\Sigma: \quad \begin{aligned} \dot{\zeta}_1 &= \zeta_2, \cdots, \dot{\zeta}_{r-1} = \zeta_r \\ \dot{\zeta}_r &= \alpha(\zeta, \eta) + \beta(\zeta, \eta)u \\ \dot{\eta} &= s(\zeta, \eta, u) \\ y &= \zeta_1, \qquad \zeta \in \mathbb{R}^r, \eta \in \mathbb{R}^{n-r}. \end{aligned} \tag{3}$$

Here, the vector $\zeta = \left[y, \cdots, y^{(r-1)}\right]^T$ consists of the system's output and its $r-1$ time derivatives which are independent of the control variable. The number r represents the well defined relative degree of the system and describes after how many time derivatives the control variable can be seen from the system's output. To reproduce the reference y^d the nominal control variable has to be chosen as

$$u^d = \frac{1}{\beta(\zeta^d, \eta)}(\dot{\zeta}_r^d - \alpha(\zeta^d, \eta)) \tag{4}$$

where $\zeta^d = \left[y^d, \cdots, y^{d^{(r-1)}}\right]^T$ contains the reference variable and its time derivatives. All signals in ζ^d have to be bounded ($y^d \in \mathcal{C}^{r-1}$) to guarantee a bounded control signal. Applying the control law (4) to (3) results in an integrator chain driven by the desired time derivative $y^{d^{(r)}}$ and an unobservable dynamics

$$\dot{\eta} = s(\zeta^d, \eta, u^d) \tag{5}$$

which is also called the *zero dynamics* or internal dynamics. In the case of a stable subsystem (5) the system (1) is called minimum phase and in the case of unstable zero dynamics the system (1) is called nonminimum phase. It is obvious that stable invertibility of the original system does not exist for nonminimum phase systems due to internal instability.
The nominal state trajectory is given by the inverse state transformation

$$x^d = \Phi^{-1}(\zeta^d, \eta). \tag{6}$$

It follows from (6) that the initial state $x^d(t=0)$ to reproduce $y^d(t=0)$ represents an $n-r$ dimensional hyperplane in the n dimensional Euclidian space.

3.2 Nonminimum phase systems

For systems with unstable zero dynamics a minimum phase approximation of the original system has to be determined and inverted. A survey of approximate inversion strategies can be found in [6]. One possible approach is to consider the standard strategy for linear systems. The stable invertible approximation of the original system is typically given by factoring the original system in a minimum phase factor and an all-pass factor. There are limited results for the factorisation of general nonlinear systems at the moment [7]. However, applicable solutions for restricted classes of problems can be found. For second order systems [8] and involutive systems [9] (the state space exact linearisation problem is solvable [1]) constructive design procedures exist to derive a nonlinear output map for the original state dynamics which yields the same steady-state locus but which has stable zero dynamics. For maximum phase systems [10], slightly nonminimum phase systems (unstable part of the internal dynamics (5) much faster than the stable part) [11], systems with fast zero dynamics (zero dynamics include only fast stable and fast unstable components in comparison to the state dynamics) [6] a minimum phase approximation can be derived without direct redefinition of a new output function. The decision for any given case is usually made by observing the linearisation of (1) along the interesting part of the steady-state locus.
The design procedure for maximum phase systems [10] is described in more detail due to its application to the example. A dynamic system is maximum phase if there are no stable or centre manifolds in the unforced system's zero dynamics. For linear systems, this corresponds to all zeros being located in the closed right-half plane (RHP). A minimum phase factor can be derived for such systems with the following properties along the steady-state locus:
1) preserved steady state locus
2) unchanged unforced dynamics
3) the zeros of the linearisation are located in the left-half plane and correspond to the reflection about the imaginary axis of the RHP zeros of the linearisation of the original system
4) identical poles of its linearisation and the linearisation of the original system.

To construct such a factor the system (1) is transformed by the local diffeomorphism $\boldsymbol{\zeta} = \boldsymbol{\Phi}(\boldsymbol{x}, u, \dot{u}, \cdots, u^{(n-r-1)})$ into Fliess canonical form:

$$\Sigma: \quad \begin{aligned} \dot{\zeta}_1 &= \zeta_2, \cdots, \dot{\zeta}_{n-1} = \zeta_n \\ \dot{\zeta}_n &= F(\boldsymbol{\zeta}, u, \dot{u}, \cdots, u^{(n-r)}) \\ y &= \zeta_1. \end{aligned} \tag{7}$$

Here, the new state variables are given by the first $n-1$ time derivatives of the system's output. The linearisation of (7) and subsequent Laplace transformation gives the following transfer function

$$\begin{aligned} G(s) &= \frac{y(s)}{u(s)} = \frac{b_{n-r}s^{n-r} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0} \\ a_i &= -\left.\frac{\partial F}{\partial \zeta_{i+1}}\right|_0, b_i = \left.\frac{\partial F}{\partial u^{(i)}}\right|_0. \end{aligned} \tag{8}$$

There are now several modifications of the Fliess form which have the beneficial effect that the "pole dynamics" can be retained whilst the zero dynamics can be merely altered. Numerator and denominator of the transfer function appear separately in the linearisation of (7). The required minimum phase factor can be constructed from the original system by substituting the value $u^{(i)}$ in the Fliess canonical form (8) for $(-1)^i u^{(i)}$ [10]. Then, the minimum phase approximation is given by

$$\hat{\Sigma}: \quad \begin{aligned} \dot{\hat{\zeta}}_1 &= \hat{\zeta}_2, \cdots, \dot{\hat{\zeta}}_{n-1} = \hat{\zeta}_n \\ \dot{\hat{\zeta}}_n &= F(\hat{\boldsymbol{\zeta}}, \hat{u}, -\dot{\hat{u}}, \cdots, (-1)^{n-r}\hat{u}^{(n-r)}) \\ \hat{y} &= \hat{\zeta}_1. \end{aligned} \tag{9}$$

Here, to distinguish from the original system, all variables are denoted by ̂. The approximated inversion has to be carried out using this state space model in non minimal realisation (depending on the derivatives of u). By setting the control variable and its first $n-r-1$ time derivatives equal to the states of the zero dynamics and assuming that the new control variable is $\hat{u}_{new} = \hat{u}^{(n-r)}$ we obtain a system in the form of (3):

$$\hat{\Sigma}: \quad \begin{aligned} \dot{\hat{\zeta}}_1 &= \hat{\zeta}_2, \cdots, \dot{\hat{\zeta}}_{n-1} = \hat{\zeta}_n \\ \dot{\hat{\zeta}}_n &= F(\hat{\boldsymbol{\zeta}}, \hat{\eta}_1, -\hat{\eta}_2, \cdots, (-1)^{n-r-1}\hat{u}_{new}) \\ \dot{\hat{\eta}}_1 &= \hat{\eta}_2, \cdots, \dot{\hat{\eta}}_{n-r} = \hat{u}_{new} \\ \hat{y} &= \hat{\zeta}_1. \end{aligned} \tag{10}$$

After this step the inversion can be done following the standard procedure under the condition that $\hat{y}^d = y^d$. The control signal finally applied is given by $\hat{u}^d = \hat{\eta}_1$. An approximation of the tracking trajectory of the state is calculated by

$$\hat{\boldsymbol{x}}^d = \boldsymbol{\Phi}^{-1}(\hat{\boldsymbol{\zeta}}^d, \hat{\eta}_1, -\hat{\eta}_2, \cdots, (-1)^{n-r-1}\hat{\eta}_{n-r}). \tag{11}$$

4 Stabilising Feedback Controller

For simplicity we assume that for nonminimum phase systems an approximation with stable zero dynamics can be obtained providing the same structure and number of states as the original system (1). Such a representation does not explicitly have to exist, as will be seen later. Then, the inversion problem for this fictitious plant can be written as

$$\begin{aligned} \dot{\hat{\mathrm{x}}}^d &= \hat{f}(\hat{\boldsymbol{x}}^d) + \hat{g}(\hat{\boldsymbol{x}}^d)\hat{u}^d \\ \hat{y}^d &= \hat{h}(\hat{\boldsymbol{x}}^d) \end{aligned} \tag{12}$$

where $\hat{f}$, $\hat{g}$ and $\hat{h}$ are approximations of the original functions in (1). In the case of a *good* system approximation the state and output trajectories of (1) and (12) have to be close for the same initial conditions and input signals. To design the controller a model of the tracking error of the states and the output is to be created. This can be straightforwardly worked out from the simplified control scheme in fig. 2. There, the inversion block is neglected and only its outputs are represented as external inputs. Additionally, an integrator was added to the state space model to achieve static exactness for output tracking. Hence, the nonlinear error model is given by

$$\begin{aligned} \begin{pmatrix} \dot{\boldsymbol{x}}_e \\ \dot{\sigma} \end{pmatrix} &= \begin{pmatrix} \dot{\boldsymbol{x}} - \dot{\hat{\boldsymbol{x}}}^d \\ y - \hat{y}^d \end{pmatrix} \\ &= \begin{pmatrix} f(\boldsymbol{x}) + g(\boldsymbol{x})u - (\hat{f}(\hat{\boldsymbol{x}}^d) + \hat{g}(\hat{\boldsymbol{x}}^d)\hat{u}^d) \\ h(\boldsymbol{x}) - \hat{h}(\hat{\boldsymbol{x}}^d) \end{pmatrix} \end{aligned} \tag{13}$$

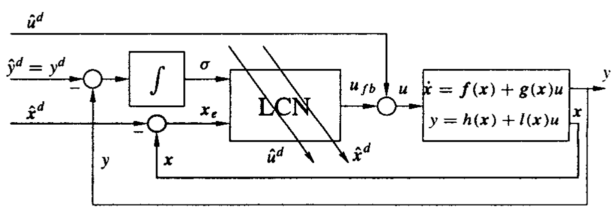

Figure 2: Closed loop system with integrator

To establish a controller design the nonlinear error model will be linearised along the approximated nominal trajectories $\hat{\boldsymbol{x}}^d$ and $\hat{u}^d$. The linearisation yields under consideration of $u = \hat{u}^d + u_{fb}$ the following linear time varying (LTV) or to be precise linear parameter varying (LPV) system:

$$\begin{aligned} \begin{pmatrix} \dot{\boldsymbol{x}}_e \\ \dot{\sigma} \end{pmatrix} &= \underbrace{\begin{pmatrix} \boldsymbol{A}(\hat{\boldsymbol{x}}^d, \hat{u}^d) & \boldsymbol{0} \\ \boldsymbol{c}^T(\hat{\boldsymbol{x}}^d, \hat{u}^d) & 0 \end{pmatrix}}_{\tilde{\boldsymbol{A}}(\hat{\boldsymbol{x}}^d, \hat{u}^d)} \begin{pmatrix} \boldsymbol{x}_e \\ \sigma \end{pmatrix} + \underbrace{\begin{pmatrix} \boldsymbol{b}(\hat{\boldsymbol{x}}^d) \\ 0 \end{pmatrix}}_{\tilde{\boldsymbol{b}}(\hat{\boldsymbol{x}}^d)} u_{fb} \\ &\quad + \begin{pmatrix} z_1(\hat{\boldsymbol{x}}^d, \hat{u}^d) \\ z_2(\hat{\boldsymbol{x}}^d, \hat{u}^d) \end{pmatrix} + \text{higher order terms.} \end{aligned} \tag{14}$$

Here, the linear parameter dependent matrix $\boldsymbol{A}(\hat{\boldsymbol{x}}^d, \hat{u}^d)$ and the linear vectors $\boldsymbol{b}(\hat{\boldsymbol{x}}^d)$, $\boldsymbol{c}^T(\hat{\boldsymbol{x}}^d, \hat{u}^d)$ result from the Jacobian linearisation of (1). The parameters consist of the outputs of the inversion $\hat{\boldsymbol{x}}^d$ and $\hat{u}^d$. Differences in the state space models (1) and (2) cause the nonlinear disturbances z_1 and z_2, which vanish in the case of minimum phase systems (exact inversion). The error model (14) is only valid if the system state is close to the desired trajectories and accordingly if the tracking error is small. However, the major nonlinearities are already captured by the nonlinear control law (4). In order to reduce the tracking error, the LPV error model (14) has to be stabilised for all possible parameter trajectories. This means at least to realise a bounded tracking error ($\mathcal{L}_2$-stability). The output will be exactly tracked for constant references if asymptotic stability of the controlled LPV-system (14) can be achieved for constant parameters in (14). In general, the bandwidth of the controlled error model has to be specified larger than the bandwidth of the reference model Σ_{ref} to counteract changes in the reference signals quickly enough.

The stabilisation of the LPV-system (14) has been established using a special gain-scheduling approach, called a local controller network [5]. A set of N linear error models with fixed parameters forms the basis for the controller design:

$$\begin{pmatrix} \dot{x}_e \\ \dot{\sigma} \end{pmatrix} = \tilde{A}(\hat{x}_i^d, \hat{u}_i^d) \begin{pmatrix} x_e \\ \sigma \end{pmatrix} + \tilde{b}(\hat{x}_i^d) u_{fb_i} \qquad i = 1, \ldots, N \tag{15}$$

Due to the missing information about the future trajectories the parameters are chosen as points of a grid covering the interesting part of the parameter space. Each model describes the error model (14) for situations when the trajectories are close to the corresponding parameter point. A simple state feedback $u_{fb,i} = -k_i^T[x_e, \sigma]^T$ is designed by pole-placement for all N models. The local controllers are combined to the final nonlinear controller by a smooth interpolation:

$$u_{fb} = -\sum_{i=1}^{N} \psi_i(\hat{x}^d, \hat{u}^d) k_i^T \begin{pmatrix} x_e \\ \sigma \end{pmatrix} \tag{16}$$

Here, the validity functions $\psi_i(\hat{x}^d, \hat{u}^d), i = 1, \ldots, N$ weight the local controller outputs in dependence on the ability of the local model to describe the error model for a given parameter set $(\hat{x}^d, \hat{u}^d)$. This results in the fact that only local controllers which are designed in the parameter space close to the present point are active. For all values of the parameters the total validity of the local models or controllers has to be one. A validity function ψ_i is normally so defined that its value is close to one near the corresponding point $(\hat{x}_i^d, \hat{u}_i^d)$. We employed normalised Gaussian functions as validity functions to get the desired behaviour.

5 Bioreactor Control

5.1 Plant description

The new approach is demonstrated on a bioreactor. A continuous-time mathematical model of the bioreactor, as described in [12], is given by the following nonlinear differential equations

$$\begin{aligned} \dot{x}_1 &= -x_1 u + x_1(1 - x_2)e^{\frac{x_2}{\Gamma}} \\ \dot{x}_2 &= -x_2 u + x_1(1 - x_2)e^{\frac{x_2}{\Gamma}} \frac{1+\beta}{1+\beta - x_2} \\ y &= x_1. \end{aligned} \tag{17}$$

The system consists of a reactor tank, where nutrients are fed in with the flowrate u. The level of the reactor is kept constant by setting the outflow equal to the inflow, $u = u_{in} = u_{out}$. Here, x_1 is the concentration of biological cells in the tank, $x_2 = (S_F - S)/S_F$ is the nutrient concentration, where S_F is the concentration of substrate in the fed to the reactor, and S is the concentration of substrate in the reactor. The parameters of the reactor are the growth rate of the cells β and the rate of nutrient consumption Γ. These constants are set nominally to $\beta = 0.02$ and $\Gamma = 0.48$ in. The flowrate u is manipulated in order to achieve a desired yield of biological cells x_1.

5.2 Control results

The process has a non-unique inverse. This results in the fact that a given cell concentration is attainable by two distinct values of the control signal and nutrition concentration. For the reason of technical relevance the reactor is always operated at the operating points related to the higher nutrition concentration. Consequently, the interesting static operation points have to be located in the region $0.02 \leq x_1^e \leq 0.255$ and $0.52 \leq x_2^e \leq 1$. Here, the index ^{e} denotes the equilibrium. In this region, the bioreactor shows nonminimum phase behaviour and is also unstable for equilibrium points with a cell concentration larger than $x_1^e > 0.1355$. The proposed control strategy was applied to the reactor. We have chosen a linear second order reference model to specify the desired tracking performance

$$\Sigma_{ref} : \frac{y^d(s)}{y^r(s)} = G_{ref}(s) = \frac{0.7^2}{s^2 + 2 \cdot 0.7s + 0.7^2}. \tag{18}$$

The bioreactor (17) is already given in Byrnes-Isidori normal form with a relative degree $r = 1$ and first order unstable zero dynamics. Consequently, this process represents a maximum phase system and can be inverted by the described procedure. The design of the LCN was carried out using 495 linear models at the points of a grid in the three dimensional parameter space of the error model for the reactor. This large number was necessary due to the widely changing dynamics of (14) for different parameters. The poles for the controlled linear models have been placed at $p_1 = -5$, $p_2 = -4$ and $p_3 = -3$. With the same performance settings, it was not possible to determine a single state feedback controller instead of the LCN. To evaluate the robustness of this control strategy, the bioreactor with perturbed system parameters ($\Gamma = 0.456$, $\beta = 0.016$) was controlled in a simulation. The results are shown in fig. 3 and 4. Apart from the remaining "inverse response behaviour", the cell concentration (solid line) follows the desired trajectory (dashed) nearly perfectly.

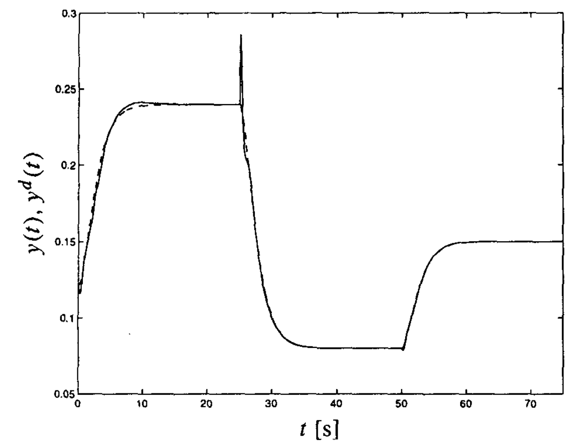

Figure 3: Trajectory of the cell concentration (solid) and desired cell concentration (dashed) under the condition of altered plant parameters

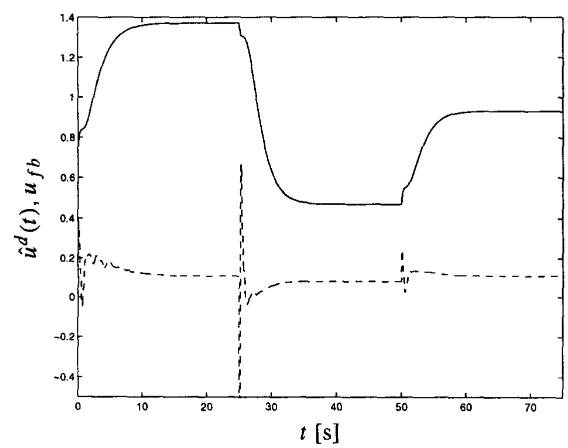

Figure 4: Trajectory of the approximated nominal control signal (solid) and the additional control signal (dashed) which is generated by the LCN under the condition of altered plant parameters

6 Conclusions

A control strategy for the tracking problem of nonminimum phase nonlinear plants was presented and successfully applied to a biological process application. The good tracking performance is achieved by an inversion of the plant using geometric methods. Placing the inverse system outside the closed loop as a feedforward controller increases the robustness against model changes in comparison to the standard input-output linearisation. A gain scheduling controller based on linear methods in the feedback loop successfully reduces the tracking error which results from a non perfect cancellation of the plant dynamics, attenuates external disturbances and stabilises possibly open-loop unstable plants. While significant effort is required to design the feedforward part of the control structure, the determination of the feedback controller is straightforward.

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