Exercise 2

Exercise 2.1

Consider the Petri nets given in Figure 1. Analyze each Petri net regarding its boundedness, liveness, and persistence properties.

Another potential feature of a Petri net is reversibility. A Petri net is called reversible if the initial marking $x^0$ is reachable from all possible $x \in R(N, x^0)$. Are the Petri nets in Figure 1 reversible?

![Petri nets for Exercise 2.1](image)

Figure 1: Petri nets for Exercise 2.1.
Exercise 2.2
The Petri net in Figure 2 is given.

Figure 2: Petri net of Exercise 2.2.

a) State the incidence matrix of the Petri net. Does the incidence matrix provide all information on the Petri net graph? Explain your answer.

b) Construct the coverability tree for the Petri net.

c) Is the Petri net
   • bounded?
   • reversible?
   • persistent?

   What can be said about the liveness of each transition?

d) A Petri net is called blocking if a state $x \in R(N, x^0)$ exists in which no transition can fire. Is the Petri net blocking? Justify.

e) Is the Petri net conservative with respect to the vector $\gamma_1 = [1 1 0 1]^t$? How about $\gamma_2 = [1 1 0 0]^t$?

Exercise 2.3
The Petri net in Figure 3 is given.

Figure 3: Petri net of Exercise 2.3.
a) Construct the coverability tree for this Petri net.

b) Are the following states reachable? Check possible sequences of transitions in the Petri net, if no statement can be made based on the coverability tree.

\[
\begin{align*}
\xi_1 &= \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, & \xi_2 &= \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, & \xi_3 &= \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, & \xi_4 &= \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\end{align*}
\]

c) Determine – if possible – a sequence of transitions that can be generated by the coverability tree, but not by the Petri net.

d) Check if the Petri net is conservative with respect to the following vectors: 
\(
\gamma_1 = [2 \ 0 \ 2 \ 0 \ 1]', \\
\gamma_2 = [1 \ 0 \ 1 \ 0 \ 0]', \\
\gamma_3 = [0 \ 1 \ 0 \ 0 \ -1]'.
\) Is it possible to decide based on the coverability tree?

Exercise 2.4

Consider the Petri net shown in Figure 4.

![Petri net of Exercise 2.4](image)

Figure 4: Petri net of Exercise 2.4.

a) Evaluate the liveness of each transition.

b) Construct the coverability tree for the Petri net.

c) Is the Petri net bounded? Justify your answer based on the coverability tree.

d) Is the Petri net conservative with respect to the following vectors?

\[
\begin{align*}
\gamma_1 &= \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, & \gamma_2 &= \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, & \gamma_3 &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\end{align*}
\]

e) Repeat items a) to d) for the initial marking 
\(\mathbf{x}^0 = [0 \ 1 \ 1 \ 1 \ 0]'\).

f) Find an initial marking for which transition \(t_6\) is L3-live but not live. For this initial marking, is the Petri net blocking?