“Optimal control of the unicycle and anthropomorphic image reconstruction”

Fix two points-directions \((x_1, y_1, \theta_1)\) and \((x_2, y_2, \theta_2)\) on the plane. We want to find a curve starting from \((x_1, y_1)\) with direction \(\theta_1\) and ending in \((x_2, y_2)\) with direction \(\theta_2\) minimizing a compromise between length and curvature.

We rewrite this problem as a problem of optimal control of the unicycle. If the direction is considered with orientation we find the Dubin’s car, if instead we consider direction regardless to orientation we find the Reeds-Shepp car (in both cases we have no constraint on curvature).

In the first case, we find that the problem has no minimum for some boundary condition \((x_1, y_1, \theta_1)\), \((x_2, y_2, \theta_1)\). In the second case, we always have existence, but minimizers may have cusps.

The second problem also arises in a model of visual perception by Petitot-Citti-Sarti. We use it to propose two anthropomorphic algorithms for image reconstruction. In the first algorithm we use minimizing curves to connect different lines of the same level set. In the second we define a diffusion process related with the dynamics of the unicycle and we reconstruct the image computing its evolution with respect to the diffusion.