Examples of invariant tracking control for the kinematic car: planar and spherical case

The notion of symmetry of a dynamical system has been a subject of long standing interest in the treatment of dynamical systems. Roughly speaking, a transformation (or a family of transformations) is a symmetry (group) of a dynamical system if the transformation maps solutions to solutions. The knowledge of admitted symmetries of a control system can help to obtain a qualitative understanding of the dynamics and the underlying problem that has been modeled [1].

The model of the kinematic car driving in a plane, also known as unicycle, is one of the most prominent examples in nonlinear control. In this contribution it serves as an example for a control system for which the associated tracking control problem naturally enjoys relevant symmetry properties, i.e. invariance w.r.t. translation and rotation of the car (actions of SE(2)). In general, symmetry properties are not invariant w.r.t. feedback. Consequently, symmetries may be lost if feedbacks are designed based on tracking errors which are incompatible with the identified symmetry. This motivates the design of compatible feedback laws, denoted as invariant feedbacks, based on invariant tracking errors as proposed in [2, 3].

In the planar case, basic geometric considerations yield an invariant tracking error given by a contouring error and a perpendicular distance w.r.t. the reference trajectory. Due to the symmetry of the model equations, the model can also be stated in a coordinate-free fashion as a left-invariant system on SE(2). Using a Lie group framework, the motion of the kinematic car on an embedded sphere $S^2$ can be described by a left-invariant system on SO(3). Further, the design of an invariant tracking control can be carried out by exploiting the Lie group structure of the problem.

References