LINEAR CONTROLLER DESIGN FOR THE SINGLE LIMB MOVEMENT OF PARAPLEGICS

Thomas Schauer * Kenneth J. Hunt *

* Centre for Systems and Control, Department of Mechanical Engineering, University of Glasgow, Glasgow G12 8QQ, Scotland
E-mail: {t.schauer,k.hunt}@mech.gla.ac.uk

Abstract: This paper deals with the problem of modelling and control of the knee joint dynamics. Using controlled functional electrical stimulation (FES) of the quadriceps muscle group the knee joint will be positioned at a commanded angle, subject to disturbances and displacements of the hip and ankle joints. A simple ARX-model has been estimated directly from measured input-output plant data. Further, a discrete linear pole-placement controller will be presented as solution for the tracking and regulation problems. Advantages of this approach in comparison with nonlinear controllers based on physiological models will be discussed and natural limitations are shown. A simple tuning rule for an already determined controller will be given to achieve robust stability with respect to the plant characteristics, which depend on the actual operating point. Experiments with neurologically intact subjects show encouraging results.

Keywords: Biocybernetics, Electrical stimulation, Pole assignment, Discrete time, Parameter estimation

1. INTRODUCTION

The control of single limb movements of paraplegics represents an important preliminary stage towards more complex motor function restoration of handicapped persons suffering from spinal cord injury. Of particular interest in this case are tasks like standing-up, standing and sitting-down. Using functional electrical stimulation (FES) with surface electrodes results in muscle contraction and subsequent joint movement. A feedback control system will be investigated for knee joint movement. Here, the quadriceps muscles (rectus femoris and vasti) are stimulated by one pair of electrodes. The resulting knee joint angle is measured. Although this experiment seems simple, it is a good example in the study of the general characteristics of human kinematics. Because of the relatively large movement range a non-isometric muscle contraction arises, so that the moment generated at the joint depends very strongly on the muscle length and velocity. Both of these variables are functions of the set of joint angles and their derivatives. Movements of the hip joint for example have an influence on the knee joint due to the biarticular behaviour of the rectus femoris muscle group. Additional complicating effects are the presence of a nonlinear recruitment curve for the muscle activation as well as some nonlinear terms in the equations of motion due to gravity and elastic passive moment. Constraints on the input (stimulation level) and output (limited range of knee angle) make the controller design more difficult.

To deal with this problem several authors have developed neuromusculoskeletal models and then used for controller design (Palazzo et al., 1998; Riener and Quintern, 1996; Riener and Fuhr, 1998; Ježernik and Riener, 1999; Chizeck et al., 1999). These models take into account the major properties of the muscle and segmental dynamics.
A simple discrete-time ARX-model is estimated directly from measured input-output plant data. The model is described by

\[ y(k) = \frac{B(q^{-1})q^{-n_s}}{A(q^{-1})}u(k) + v(k) \quad (1) \]


2.2 Controller Design

Based on the identified model a linear discrete input-output pole-placement controller with two degrees of freedom (2DOF) can be designed (Åström and Wittenmark, 1997) for the selected operating point. The controller has the general form (see Fig. 2)

\[ u(k) = \frac{1}{R} (T_r(k) - S(y(k) + n(k))). \]  

(4)

Here, \( r(k) \) is the reference signal and \( n(k) \) a measurement noise. \( R \) and \( S \) are the controller polynomials in the delay operator, which are defined by

\[ R(q^{-1}) = 1 + r_1 q^{-1} + \cdots + r_m q^{-m} \]  

(5)

\[ S(q^{-1}) = s_0 + s_1 q^{-1} + \cdots + s_n q^{-n}. \]  

(6)

These polynomials and the pulse transfer function \( T \) have to be determined in such a way that the desired output response \( y_m \) to command signals \( r \) becomes:

\[ y_m(k) = H_m(q^{-1})r(k) = \frac{B_m(q^{-1})}{A_m(q^{-1})}r(k). \]  

(7)

It is further assumed that the controller can cancel some of the plant poles and zeros. Assume that the polynomials \( A \) and \( B \) are factorized as \( A = A^+ A^- \) and \( B = B^+ B^- \) where \( A^+ \) and \( B^+ \) are the stable factors that will be cancelled. To obtain perfect model following the numerator \( B_m \) of the reference model \( H_m \) must contain the factor \( q^{-n_k} B^- \), because \( q^{-n_k} B^- \) cannot be cancelled. Thus, \( B_m \) can be written as \( B_m = q^{-n_k} B^- T_m \). For the attenuation of constant disturbances the controller is required to have integral action. This means that the controller polynomial \( R \) has to contain the factor \((1 - q^{-1})\). The above requirements and assumptions give

\[ R = (1 - q^{-1}) B^+ T \]  

(8)

\[ S = A^+ \overline{S} \]  

(9)

\[ T = \frac{\overline{B_m} A_o A^+}{A_m} \]  

(10)

where the polynomial \( A_o \) is denoted as the observer polynomial. Combining (1)-(10), the closed-loop characteristic polynomial \( A_{cl} \) is easily found by

\[ A_{cl} = A R + q^{-n_k} B S = A^+ B^+ A_o \]

\[ A_{cl} = A^+ B^+ (A^- (1 - q^{-1}) \overline{R} + q^{-n_k} B^- \overline{S}) \]

\[ = A^+ B^+ A_o. \]  

(11)

To obtain the controller the Diophantine equation (11) has to be solved for \( \overline{R}, \overline{S}, T \) computed from equations (8)-(10). The specifications
for tracking are governed by the pulse transfer function $H_m = B_m/A_m$. The desired regulation behaviour is given by the observer polynomial $A_o$. Separation of the disturbance and command signal response is achieved. To see this the transfer functions from the command signal $r$, the output disturbance $v$ and the measurement noise $n$ to the output $y$ are calculated as

$$y = \frac{q^{-n_k} BT}{AR + q^{-n_k} BS} r + \frac{AR}{AR + q^{-n_k} BS} v - \frac{q^{-n_k} BS}{AR + q^{-n_k} BS} n$$

$$y = \frac{B_m}{A_m} r + \frac{(1 - q^{-1}) A^{-1} T}{A_o} v - \frac{q^{-n_k} B^{-1} S}{A_o} n.$$  \hspace{1cm} (12)

The control structure is shown in Fig. 2. Here, the constant gain $k_p$ will be explained later and can be assumed as $k_p = 1$ at the moment. To achieve unity gain of the reference model and to leave the system zeros unchanged ($B^- = B, B^+ = 1$) the polynomial $B_m$ is chosen as

$$B_m = q^{-n_k} B A_m(1)/B(1).$$ \hspace{1cm} (13)

The plant characteristics are dependent upon the equilibrium point. In summary, the system poles for low knee extension are complex conjugate and underdamped and become real and overdamped for near-full knee extension. The system gain decreases by a factor of approximately 5-15 in the same range of operation. The significant change in

$$y(k) = \frac{0.0005534 q^{-2}}{1 - 1.21 q^{-1} + 0.41 q^{-2}} u(k) + v(k).$$ \hspace{1cm} (14)

3. RESULTS AND DISCUSSION

In the following experimental results with a neurologically intact subject will be presented. The local model and controller are designed for the static operating point $\Theta = 0.7$ and $u^* = 150 \mu s$, corresponding to a nearly extended knee. For the plant model, obtained from the identification trials, the structure is given by $n_a = 2$, $n_b = 0$ and $n_k = 2$. Fig. 3 shows data from the identification and from the identified model, given by the following pulse transfer function:

The system gain is critical for controller design. A controller designed for an equilibrium point with smaller gain can destabilise the closed loop during operation at an equilibrium point with larger plant gain. To achieve a controller with stability robustness it is recommended to use the linear model with the largest gain for controller design. Another possibility is to adapt the open-loop gain to compensate the changing system gain. This procedure works if a controller has already been designed. In both cases it is assumed that the system time constants do not vary widely for different operating points. Adapting the open-loop gain means that the factor $k_p$ in Fig. 2 will be changed. Results of control experiments are shown in Fig. 4 and 5. Here, the controller C1, as well as the later modification, is based on the model (14). To specify the polynomials $A_m$ and $A_o$ with degree deg $A_m = \deg A_o = 2$ discretisation of a continuous time system of 2nd order was used. This system is defined by the rise time $t_{rA_m}$ or $t_{rA_o}$ and the damping coefficient $\zeta_{A_m}$ or $\zeta_{A_o}$. For all controllers the design parameters are shown in Table 1. The control signal is relatively smooth and the knee angle of the subject follows the filtered reference angle $y_m$ very accurately. Such a control signal is desirable for paraplegic patients, because a smooth control signal is less likely to excite unwanted spastic reflexes. Although the controller was designed for the knee angle range $\Theta = 0.6 \ldots 0.8$ it works very well in the range $\Theta = 0.3 \ldots 0.9$ (a range of approximate 60°). Due
to the characteristics of the muscle groups during FES it was impossible to reach an angle higher than \( \Theta = 0.9 \). For angles \( \Theta < 0.3 \) stability could not be achieved (without loss of performance). The controller is also able to counteract disturbances successfully (cf. Fig. 5). The leg was disturbed by pushing it downwards at \( t = 4 \) and \( t = 13s \) and by lifting it up at \( t = 7.5s \) and \( t = 18s \). At \( t = 20s \) the controller goes into saturation but comes back at \( t = 21s \) because of the implemented antiwindup scheme (Åström and Wittenmark, 1997). Further, the subject was allowed to alter hip angle slightly during the experiment. Changing the rise time of the reference model to \( t_{rA_m} = 0.5s \) (controller C2), the results shown in Fig. 6 were obtained. Now the closed-loop system becomes unstable for the equilibrium point \( \Theta = 0.3 \) because the system gain is too high with the fast controller specifications. Changing the open-loop gain as suggested above by the factor \( k_p = 0.5 \) (controller C3) stabilises the system but gives poor performance for upward movement of the leg (cf. Fig. 7). In this experiment the upper saturation limit in the controller was set to \( u = 250\mu s \) so that the reference value for the angle \( r = 0.9 \) could not be reached. At \( t = 13s \) the leg was lifted up as disturbance test.

Table 1 Controller design parameters

<table>
<thead>
<tr>
<th>Controller</th>
<th>( t_{rA_m} )</th>
<th>( \zeta_{A_m} )</th>
<th>( t_{rA_o} )</th>
<th>( \zeta_{A_o} )</th>
<th>( k_p )</th>
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<td>1</td>
<td>1</td>
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</tr>
<tr>
<td>C2</td>
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<td>0.4s</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>C3</td>
<td>0.5s</td>
<td>0.4s</td>
<td>1</td>
<td>0.5</td>
<td></td>
</tr>
</tbody>
</table>

4. CONCLUSIONS

It was shown that good tracking performance and disturbance attenuation for the knee joint movement can be achieved by controlled FES using...
surface electrodes. The use of a parameterized linear model and pole-placement design provides a fast way to determine a simple but reliable and robust controller. Closed-loop specifications can be easily formulated in the time-domain and are model independent. There are plans to continue this investigation with experiments on paraplegic subjects. The integration of this approach into problems like standing with a loaded knee joint (Wood et al., 1998) will be investigated in the near future.

5. REFERENCES


