Exercise 1 — Solution

Exercise 1.1

a) A possible Petri net model for the dental clinic is shown below.

\[
\begin{align*}
A^- &= \begin{bmatrix}
0 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0
\end{bmatrix},
A^+ &= \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\end{align*}
\]

The question whether it is possible for a medical examination to start translates to whether it is possible to fire \(t_4\); we know (see lecture notes, page 16) that \(t_4\) can fire at \(x(5)\) if and only if \(x(5) \geq A^- u_4 = [1 \ 0 \ 1 \ 0 \ 1]'\).

In order to obtain \(x(5)\), we simply apply \(x(k+1) = x(k) + Au_j\) for \(k = 0, 1, \ldots, 4\) according to the observed firings, starting with \(x(0) = x^0 = [0 \ 0 \ 2 \ 0 \ 1]'\). We can assume, without loss of
generality, that the three firings of \( t_1 \) have occurred in a row in the beginning, followed by the two firings of \( t_2 \). We then have

\[
\begin{align*}
  x(1) &= x(0) + A u_1 \\
  x(2) &= x(1) + A u_1 = x(0) + 2A u_1 \\
  x(3) &= x(2) + A u_1 = x(0) + 3A u_1 \\
  x(4) &= x(3) + A u_2 = x(0) + 3A u_1 + A u_2 \\
  x(5) &= x(4) + A u_2 = x(0) + 3A u_1 + 2A u_2 = [1 \ 2 \ 0 \ 0 \ 1]' \\
\end{align*}
\]

One can easily see that \( x(5) \nleq A^{-1} u_4 \), so the medical examination cannot start.

c) The modified Petri net model is shown below.

![Modified Petri Net Model](image)

d) The new Petri net model is shown below.

![New Petri Net Model](image)
Exercise 1.2

a) A possible Petri net model for the workshop is shown below.

```
1  -1  0  0
0  1 -1  0
0  0  1 -1
-1  1  0  0
0  -1  1  0
0  0  -1  1
```

b)

c) The reachability graph is shown below.

Exercise 1.3

a) A possible Petri net model for the traffic light system is shown below.
b)  
\[ A = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & -1 & 1 & 0 & -1 & 1 \end{bmatrix} \]

c) The reachability graph is shown below.
d) The modified Petri net model for the traffic light system is shown below.

![Diagram]

e) The Petri net from item d) is an Event Graph (see Definition 2.3 on page 18 of the Lecture Notes).