Non-conservative discrete-time ISS small-gain conditions for closed sets

This talk is concerned with a unification and a generalization of the small-gain theory subsuming a wide range of the existing small-gain theorems. In particular, we introduce small-gain conditions that are necessary and sufficient to ensure input-to-state stability (ISS) with respect to closed sets. Toward this end, we first develop a Lyapunov characterization of \( \omega \)-ISS via finite-step \( \omega \)-ISS Lyapunov functions. Then, we provide the small-gain conditions to guarantee \( \omega \)-ISS of a network of systems. Finally, applications of our results to partial input-to-state stability, ISS of time-varying systems, incremental stability and distributed observers are given.