Model-based design and experimental validation of active vibration control for a stress ribbon bridge using pneumatic muscle actuators

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A B S T R A C T

This paper describes the development of an active vibration control system for a light and flexible stress ribbon footbridge. The 13 m span carbon fiber reinforced plastic (CFRP) stress ribbon bridge was built in the laboratory of the Department of Civil and Structural Engineering, Berlin Institute of Technology. Its lightness and flexibility result in high vibration sensitivity. To reduce pedestrian-induced vibrations, very light pneumatic muscle actuators are placed at handrail level, introducing control forces. First, a reduced discretized analytical model is derived for the stress ribbon bridge. To verify the analytical prediction, experiments without feedback control are conducted. Based on this model, a delayed velocity feedback control strategy is designed. To handle the nonlinearities of the muscle actuator, a subsidiary force control is implemented. Then the control performance from numerical simulation is verified by experiments under free vibration. As a result, analytical analyses agree well with experimental results. It is demonstrated that handrail-introduced forces can efficiently control the first mode response.

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1. Introduction

Stress ribbon bridges are among the most elegant and lightest bridges. Due to their static and dynamic characteristics, they have been mainly designed for pedestrian traffic rather than for road or rail traffic [1,2]. The suspension cable and the bridge deck are combined into one stiffening element, which is anchored in the abutments. Usually, the cables are made of steel cables or steel plates. To show the potential of high-strength carbon fiber reinforced plastics (CFRPs), a 13 m span stress ribbon bridge with CFRP ribbons was built in the laboratory of the Department of Civil and Structural Engineering, Berlin Institute of Technology (TU Berlin); see Fig. 1. This composite material allows considerably smaller cross sections compared to steel. The bridge's tensile force under dead and live load is carried by only 6 ribbons with a cross section of ≈1.1 mm × 50 mm each. The combination of low extensional stiffness using CFRP for the ribbons and a lightweight bridge deck leads to considerable dynamic responses caused by pedestrian loads [3,4].

Generally, to keep vibrations within acceptable limits, several conceptual design approaches such as increasing the stiffness or increasing the dead load/traffic load ratio have been applied in practice. Sometimes, additional passive dampers have been installed to reduce high vibrations [5–8].

An alternative potential approach to ensure the structural serviceability respectively comfort criteria of footbridges is to use active vibration control (AVC). In particular, this is necessary for extremely light structures, where system properties such as the mass and stiffness become time-variant with changing pedestrian traffic. Then the natural frequencies start to depend on live loads and some passive damping techniques can no longer operate optimally.

In civil engineering structures, active vibration control has been achieved and implemented by active mass dampers (AMDs) or hybrid mass dampers (HMDs) in towers, tall buildings, and pylons of cable-stayed bridges. Controllable fluids such as electrorheological (ER) and magneto-rheological (MR) ones have been used as semi-active dampers to control wind-induced cable vibrations and buildings under earthquake excitations [9–11]. Studies on the active control of cable vibrations have been conducted in [12–14]. By axial support movements, sag-induced forces can be applied which change the cable tension and control the in-plane vibrations. Experimental studies have confirmed this control strategy, limited to the symmetric modes and efficient only for the first in-plane mode.

The applied concept of active vibration control for the stress ribbon bridge is to control the symmetric modes as well as the asymmetric modes. The natural frequencies of the CFRP...
stress ribbon bridge that coincide with the dominant frequencies of pedestrian-induced loads range from 1.34 to 3.75 Hz due to walking, running, and jumping [4]. To reduce the dynamic responses in this range, control forces are introduced at handrail level by very light pneumatic muscle actuators (PMAs) placed at the midspan and quarter points (Fig. 2a, set-up B). The handrail posts are rigidly coupled with the concrete slabs and pin-jointed with each other, except for the actuator placement. In this way, control forces can be applied along the bridge deck to produce a countermovement.

In this study, one pair of actuators mainly controlled by one sensor is considered. The actuators are placed between two handrail posts on each side at the midspan, and the sensor is placed at the midspan in the centre line (Figs. 2b, 3a and 3b). This actuator/sensor configuration (set-up A) allows control of the symmetrical modes. The control system presented here focuses only on the first symmetric mode. To design a model-based controller, an analytical control-orientated model is derived for the bridge in Section 2. The nonlinear force contraction behavior of the extremely light pneumatic muscle actuator is described in Section 3. In Section 4, modal state control strategies are investigated, and a subsidiary force control is proposed to handle the nonlinearities of the pneumatic actuator. The efficiency of the obtained control designs is tested by numerical simulations and verified by full-scale experiments focusing on free vibrations in Section 5.

### 2. Analytical model for multi-modal motions

#### 2.1. Parameters of the bridge and the derived eight-plate model

From the distributed system of the stress ribbon bridge, an analytical plane rigid body model for a multi-variable control
system is developed. In order to get good agreement with experimental data for the modes to be controlled, a plane model with seven degrees of freedom $q_i$ is developed (Fig. 4). The stiffening effect of the railing is not included, as its influence on the modes to be controlled is negligible (Table 3).

The deck of the bridge consists of six CFRP ribbons, which are covered by 16 open-jointed concrete plates in two sizes—12 long plates (0.8 m) and 4 small plates (0.6 m). To simplify the modeling, eight equal-sized plates with a length $L = 1.63$ m are discretized. The single model plates are pin-jointed and form a sprocket chain. The total mass of the real bridge including the railing is $M_T = 4336$ kg. Using uniform distribution, the weight of each model plate is

$$m_i = M_T / 8 = 542 \text{ kg} \quad (i = 1 \ldots 8). \quad (1)$$

The moment of inertia is calculated as follows:

$$I_{yy_i} = \frac{1}{12} \cdot m_i \cdot (L^2 + d^2) = 120 \text{ kg} \cdot \text{m}^2 \quad (i = 1 \ldots 8). \quad (2)$$

The parameters are listed in Table 1.

The bearing conditions at the two ends of the chain are fixed in the vertical direction and elastic in the horizontal direction. The elastic flexibility is modeled by horizontal springs. These springs represent the extensional stiffness of the CFRP ribbons, which are not modeled themselves. Hence, it can be assumed that the elastic elongation of the CFRP ribbons occurs only discrete at the bearings. The spring stiffness is calculated by the extensional stiffness $EA$ resulting from six CFRP ribbons.

$$k = EA / (L_T/2) = 7 \times 356 000 \text{ N/m}. \quad (3)$$

The effect of high elastic elongation using high-strength materials has to be taken into account for design and construction. To obtain the desired geometry under dead load, the CFRP ribbons had to be pre-stressed before they were covered with concrete plates. The state under pre-stressing without dead load is the initial state for model formulation, and it corresponds with the horizontal $w$ axis in Fig. 4. The equivalent pre-stressing force $P$ in the model is applied at each bearing.

$$P = 300 000 \text{ N}. \quad (4)$$

After the ribbons were covered with the concrete plates, the bridge got its shape due to elastic elongation. The geometric stiffness of the bridge results from the tensile force in the ribbons in this sagged state.

The structural damping is derived from measured free vibrations. As the railing has an influence on structural damping, the measurement was done under bridge set-up A. To determine the logarithmic decrement $\zeta_i$ of the $i$-th mode (Table 2), the measured signals (accelerations) were filtered. Then the modal damping ratio is calculated by $\zeta_i = \Lambda_i / 2\pi$ and the damping coefficient $C_i$ is given by

$$C_i = 2 \cdot \zeta_i \cdot M_i \cdot \omega_i. \quad (5)$$

The modal mass $M_i$ is obtained by modal transformation, as discussed in Section 3. In Eq. (5), $\omega_i$ is the natural angular frequency of the $i$-th mode of the eight-plate model (Table 3).

**Table 1** Parameters of the bridge and input parameters for the eight-plate model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bridge</th>
<th>Eight-plate model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total bridge length (m)</td>
<td>$L_T$</td>
<td>13.05</td>
</tr>
<tr>
<td>Plate length (m)</td>
<td>$L$</td>
<td>$4 \times 0.60 / 12 \times 0.80$</td>
</tr>
<tr>
<td>Plate thickness (m)</td>
<td>$d$</td>
<td>0.10</td>
</tr>
<tr>
<td>Total mass (kg)</td>
<td>$M_T$</td>
<td>4336</td>
</tr>
<tr>
<td>Plate mass (kg)</td>
<td>$m_i$</td>
<td>$4 \times 220 / 12 \times 288$</td>
</tr>
<tr>
<td>Mass moment of inertia (kgm$^2$)</td>
<td>$I_{yy_i}$</td>
<td>$4 \times 6.8 / 12 \times 15.7$</td>
</tr>
<tr>
<td>Stiffness of CFRP ribbon (MN)</td>
<td>$EA$</td>
<td>48</td>
</tr>
<tr>
<td>Spring stiffness (N/m)</td>
<td>$k$</td>
<td>7 356 000</td>
</tr>
<tr>
<td>Pre-stressing force (N)</td>
<td>$P$</td>
<td>300 000</td>
</tr>
</tbody>
</table>

**Table 2** Modal damping values of the bridge, set-up A.

<table>
<thead>
<tr>
<th>Mode</th>
<th>No. Log decrement $\Lambda_i$</th>
<th>Modal damping ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical symmetric</td>
<td>1</td>
<td>0.0136</td>
</tr>
<tr>
<td>Vertical asymmetric</td>
<td>2</td>
<td>0.0253</td>
</tr>
<tr>
<td>Vertical symmetric</td>
<td>3</td>
<td>0.0414</td>
</tr>
<tr>
<td>Vertical asymmetric</td>
<td>4</td>
<td>0.0166</td>
</tr>
<tr>
<td>Vertical symmetric</td>
<td>5</td>
<td>0.0428</td>
</tr>
<tr>
<td>Vertical asymmetric</td>
<td>6</td>
<td>0.0506</td>
</tr>
<tr>
<td>Vertical symmetric</td>
<td>7</td>
<td>0.0026</td>
</tr>
</tbody>
</table>

**2.2. Model formulation**

2.2.1. Constraints/generalized coordinates

The bridge model consists of $N = 8$ discrete masses with $p = 16$ geometric constraints. Using Eq. (6), the number $n_q$ of degrees of freedom or the generalized coordinates $q_i$ can be calculated.

$$n_q = 3N - p. \quad (6)$$

The horizontal degree of freedom can be neglected as it is decoupled from the interesting vertical movement and cannot be controlled by the actuator placement. Finally, seven vertical degrees of freedom (respectively, generalized coordinates, $q_j$) remain. For the generalized coordinates $q_j$, the pin-joint coordinates are chosen, except at the abutments. The expression for $q_j$ appears as Eq. (7), given in Box I.

The geometric relation between the elastic and geometric stiffness and the coordinates is nonlinear. Therefore, a small-angle approximation is used. The cosine function is approximated by using a Taylor series development with truncation of the series after the second term: $\cos \theta_i = 1 - 1 / 2 \theta_i^2$. Considering the constraints, the following coordinates for model formulation are used, according to Fig. 4.

$$z_i = \frac{1}{2} (\xi_i + \bar{z}) \quad (i = 1 \ldots 8) \quad (8)$$

$$\theta_i = \frac{1}{L} (\xi_i + \bar{z}) \quad (i = 1 \ldots 8) \quad (9)$$

Fig. 4. The eight-plate model (set-up A).
\( q = (q_1 \quad q_2 \quad q_3 \quad q_4 \quad q_5 \quad q_6 \quad q_7)^T = (\hat{z}_1 \quad \hat{z}_2 \quad \hat{z}_3 \quad \hat{z}_4 \quad \hat{z}_5 \quad \hat{z}_6 \quad \hat{z}_7)^T \) \hspace{1cm} (7)

Table 3

<table>
<thead>
<tr>
<th>Mode</th>
<th>No.</th>
<th>Eight-plate model</th>
<th>Bridge without railing</th>
<th>Bridge set-up A</th>
<th>Bridge set-up B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical symmetric</td>
<td>1</td>
<td>1.33</td>
<td>1.34</td>
<td>+0.8%</td>
<td>1.35</td>
</tr>
<tr>
<td>vertical asymmetric</td>
<td>2</td>
<td>2.54</td>
<td>2.48</td>
<td>−2.4%</td>
<td>2.76</td>
</tr>
<tr>
<td>vertical symmetric</td>
<td>3</td>
<td>3.85</td>
<td>3.75</td>
<td>−2.6%</td>
<td>4.08</td>
</tr>
<tr>
<td>vertical asymmetric</td>
<td>4</td>
<td>5.34</td>
<td>4.98</td>
<td></td>
<td>5.53</td>
</tr>
<tr>
<td>vertical symmetric</td>
<td>5</td>
<td>6.98</td>
<td>6.32</td>
<td></td>
<td>7.08</td>
</tr>
<tr>
<td>vertical asymmetric</td>
<td>6</td>
<td>8.67</td>
<td>7.53</td>
<td></td>
<td>8.60</td>
</tr>
<tr>
<td>vertical symmetric</td>
<td>7</td>
<td>10.09</td>
<td>8.74</td>
<td></td>
<td>9.43</td>
</tr>
</tbody>
</table>

2.2.2. Lagrange's equations of motion

Let the displacement \( q \) be the set of generalized coordinates, and force \( f \) be the generalized force. The Lagrange equations [15] can be expressed as

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_j} \right) - \frac{\partial L}{\partial q_j} = \sum Q_i \quad (j = 1 \ldots 7),
\]

where the Lagrange function

\[
L(q_j, \dot{q}_j, t) = T(q_j, \dot{q}_j, t) - V(q_j, \dot{q}_j, t) \quad (j = 1 \ldots 7),
\]

the horizontal vibration of the mass \( m_i \) is omitted because of its small influence on vertical vibrations.

\[
T = \sum_{i=1}^{8} \frac{1}{2} m_i \dot{z}_i^2 + \sum_{i=1}^{8} \frac{1}{2} \frac{1}{2} m_i \dot{w}_i^2 + \sum_{i=1}^{8} b_i y_i \dot{\theta}_i^2.
\]

The potential energy \( V \) is defined by

\[
V = \sum_{i=1}^{8} (m_i g z_i) + \frac{1}{2} k(w_i)^2 + \frac{1}{2} k(\dot{w}_i)^2.
\]

There are three types of generalized force \( Q_i \) which are considered in the Lagrange equations.

1. Conservative forces with an existing potential resulting from pre-stressing.

\[
Q_i^{(p)} = \sum_{i=1}^{8} F_i \cdot \frac{\partial r_i}{\partial q_j} = F_{u_i} \cdot \frac{\partial w_i(q_j)}{\partial q_j} + F_{\dot{w}_i} \cdot \frac{\partial \dot{w}_i(q_j)}{\partial q_j} \quad (j = 1 \ldots 7),
\]

where \( r_i \) is the vector pointing to the center of mass \( m_i \) and \( F_i \) is the force vector acting on mass \( m_i \); here,

\[
F_{u_i} = -P \quad (17)
\]

\[
F_{\dot{w}_i} = P. \quad (18)
\]

2. Non-conservative forces resulting from friction forces (respectively, damping). Damping depends on many factors such as material damping, structural damping (mechanical connections), and damping due to bearing conditions. To consider the different damping mechanisms in a model, they have to be identified. As identification is complex, damping is considered by modal damping using modal transformation, as shown in Sections 2.1 and 4.1. A phenomenological approach to consider friction forces in the Lagrange equations can be found in [15].

3. Non-conservative forces from active control introduced at handrail level in the midspan are defined by

\[
Q_i^{(AC_1)} = \sum_{i=1}^{8} F_i \cdot \frac{\partial r_i}{\partial q_j} = \sum_{i=1}^{8} \left( F_{u_i} \cdot \frac{\partial w_i(q_j)}{\partial q_j} + F_{\dot{w}_i} \cdot \frac{\partial \dot{w}_i(q_j)}{\partial q_j} \right) \quad (j = 1 \ldots 7)
\]

\[
F_{u_i} = F_{\dot{w}_i}^{(AC_1, \theta_1)} \quad (20)
\]

\[
F_{\dot{w}_i}^{(AC_1, \theta_1, h)} \quad (21)
\]

where \( AC_1 \) is the active control force generated by two pneumatic muscle actuators at the midspan (Section 4). The vertical component of \( AC_1 \) is neglected.

Using the Lagrange equations, seven nonlinear ordinary differential equations (ODEs) of second order can be derived.

2.3. Linear state-space description derived from nonlinear ODEs

The nonlinear ODEs can be linearized at the operating point (bridge without pedestrian loads) in the case of small amplitudes guaranteed by active vibration control. Therefore, the nonlinear differential equations are derived into a state-space description that is convenient for using control techniques, especially state control [16]. For the eight-plate model, seven generalized coordinates and their first derivatives are selected as state vector \( x \).

\[
x = (x_1 \quad x_2 \quad \ldots \quad x_{12})^T
\]

\[
= (q_1 \quad \dot{q}_1 \quad \ldots \quad q_7 \quad \dot{q}_7)^T.
\]

Then, the nonlinear state-space description is defined as follows:

\[
x = f(x, u) = \begin{pmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\vdots \\
\dot{x}_{13} \\
x_{14}
\end{pmatrix} = \begin{pmatrix}
f_2(x_1, \ldots, x_{14}, u) \\
\vdots \\
f_{14}(x_1, \ldots, x_{14}, u)
\end{pmatrix}.
\]
Solving the equations

\[ x = f(x_0, u_0) = 0, \]

the operating point can be calculated. Finally, the linear state-space form is obtained by determining the Jacobian matrix at the operating point.

\[ A = \left. \frac{\partial f}{\partial x} \right|_{x=x_0, u=u_0}, \]
\[ B = \left. \frac{\partial f}{\partial u} \right|_{x=x_0, u=u_0}, \]

(25)

\[ \dot{x} = Ax + Bu, \quad y = C^T x, \]

(26)

where \( A \) is a \( 14 \times 14 \) system matrix, \( x \) is a \( 14 \times 1 \) state vector, \( B \) is a \( 14 \times 1 \) input vector, \( u \) is the input variable (respectively, active control force \( AC_1 \)), \( y \) is a \( 14 \times 1 \) output vector, and \( C \) is a \( 14 \times 14 \) identity matrix.

2.4. Analysis and verification of the linear model

Simulations and experiments of free vibrations without control are conducted to verify the dynamic behavior of the model. First, the bridge without railing (Fig. 1) is compared with the eight-plate model. The first three frequencies of the analytical model agree well with the measured frequencies without the railing (Table 3). The discrepancy is smaller than 2.6 %. Second, the railing is added and actuators are placed only at the midspan (set-up A, Figs. 2b, 3a and 3b). The railing inevitably influences the natural frequencies. As only the first mode is controlled, the discrepancies of the higher frequencies can be neglected. Third, regarding multivariable control, in further research the pin-jointed bars at the quarter points will be replaced by actuators as well (set-up B, Fig. 2a). Then the handrail stiffening effects will be reduced, and finally the frequency discrepancies will decrease.

In Fig. 5, the validations of the first three modes are shown. Therefore, three cases of free vibration simulations are conducted starting from different initial conditions. For comparison, three experiments under bridge set-up A are carried out, exciting the three modes.

3. Physical model of a pneumatic muscle actuator (PMA)

Pneumatic muscle actuators (PMAs) are rather new in the field of active vibration control. They are increasingly used in the area of automation and robotics [17]. One of the major advantages is their extremely low self-weight because their core element consists of a membrane tube (Fig. 6). Thus the force/self-weight ratio is considerably higher in comparison to pneumatic cylinders and other actuators. For this reason, PMAs are used in this study.

The functionality and the force contraction behavior of a commercial pneumatic muscle actuator, a so-called fluidic muscle, which is developed and manufactured by Festo AG & Co. KG, Esslingen, Germany [18], is considered in this section. The pulling-only actuator consists of a flexible chloroprene rubber tube and an integrated rhombic grid consisting of stiff aramid fibers with connection flanges. By inflating the tube with compressed air, it expands in the radial direction nearly cylindrically and contracts in the longitudinal direction; hence only a tension force can be generated. Even though these muscles show many advantages such as high force/self-weight ratio and possible operation in rough environments, the main drawback is their nonlinear and discontinuous behavior. The contraction–pressure dependency of the force and the nonlinear contraction dependency of the volume make accurate control difficult.

Physical models to describe the force contraction characteristics have been investigated in [19]. The force \( F \) produced by a PMA mainly depends on the absolute pressure \( p \) inside the muscle and its contraction length \( s \). In the main range of application, the force can be approximated by a polynomial function of third order, as expressed by the following equation:

\[ F_{\text{eq}}(p(t), s(t)) = (p(t) - p_0) \cdot A_v - (k_1 + k_1p \cdot p(t)) \cdot s(t) - k_2 \cdot s(t)^2 - k_3 \cdot s(t)^3, \]

(27)

where \( p_0 \) is the ambient pressure, \( A_v \) the virtual muscle cross section, and \( k_i \) constant parameters, which are given in Table 3.
These parameters depend on the muscle length and muscle diameter. In this study, a muscle type DMSP-40-356N\(^\dagger\) with a diameter of 40 mm and a nominal length of 356 mm is used. Its force contraction characteristics plotted for different absolute pressure \(p\) are shown in Fig. 7. The permitted working force is 6000 N and the maximum contraction is 25 %.

4. Control design

4.1. Modal state-space representation and state estimator

To design a model-based controller for specific modes, it is convenient to use a modal state-space representation\([16]\). This representation is obtained from the nodal state-space representation by transformation of Eq. (26) using a transformation matrix, here the modal matrix, \(R\). The modal state variables \(x_m\) are introduced such that

\[
x = R \cdot x_m.
\]

Hence, the modal state-space representation can be obtained, which is characterized by the block-diagonal state matrix \(A_m\), the input matrix \(B_m\), and the output matrix \(C_m\):

\[
A_m = R^{-1}AR = \text{diag}(A_m^1, \ldots, A_m^n)
\]

\[
B_m = R^{-1}B = \begin{bmatrix}
B_m^1 \\
\vdots \\
B_m^n
\end{bmatrix},
\]

\[
C_m = CR = \begin{bmatrix}
C_m^1 & \cdots & C_m^n
\end{bmatrix}.
\]

In Eq. (29), \(\omega_i\) is the natural angular frequency of the \(i\)-th mode and \(\zeta_i\) is the modal damping ratio of the uncontrolled system according to Table 2. \(A_m^n\) is a \(2 \times 2\) matrix, \(B_m^n\) is a \(2 \times 1\) vector, and \(C_m^n\) is a \(14 \times 2\) matrix of the first mode. In the modal description, all modes are decoupled. To control the first mode, the related block components of the modal matrices are extracted. The relationship between the input \(u\) and output \(y\) is identical in modal representation and nodal representation.

Nodal state control strategies require the nodal “displacement” and “velocity” for each node of the eight-plate model. Modal state control strategies require the modal state “displacement” and its derivative “velocity” for each mode to be controlled. In practice, it is desirable to use a minimum number of sensors to measure nodal “displacements” and “velocities”. For this application, a Kalman filter is employed to estimate the modal states of the symmetric (SM) modes by measuring only the vertical displacement, state \(x_7\), in the midspan. The vertical velocity, state \(x_8\), is obtained by numerical differentiation. Observability of these modes is fulfilled. As outlined in [16], the dynamics of the estimator are given by

\[
\dot{x}^{SM}_m = A^{SM}_m \cdot \dot{x}^{SM}_m + B^{SM}_m \cdot u + L(y - C^{SM}_m \cdot \dot{x}^{SM}_m)
\]

\[
A^{SM}_m = \text{diag}(A_m^1, A_m^5, A_m^3, A_m^n)
\]

\[
B^{SM}_m = \begin{bmatrix}
B_m^1 & B_m^5 & B_m^3 & B_m^n
\end{bmatrix}^T
\]

\[
C^{SM}_m = \begin{bmatrix}
C_m^1 \\
C_m^5 \\
C_m^3 \\
C_m^n
\end{bmatrix}
\]

\[
y = \begin{bmatrix}
x_7 \\
x_8
\end{bmatrix}^T.
\]

where \(x^{SM}_m\) are the estimated symmetric modal states. The measurement is \(y\), and \(\hat{y}\) is the estimation of this measurement based on \(\dot{x}^{SM}_m\). The matrix \(C\) selects the estimated elements \(\hat{y}_i\).
and $\hat{x}_0$ out of the vector $\mathbf{e}_{m}^T \cdot \dot{\mathbf{x}}_{m}^T$. The convergence of $\mathbf{x}_{m}^T \rightarrow \mathbf{x}_{m}^T$ is determined by the estimator gain $\mathbf{L}$. This gain is found by minimizing a quadratic cost function (LQR) [16].

4.2. Modal state control

In this section, a delayed velocity feedback control strategy is presented to control the first mode vibrations. This approach is essential for the special characteristics of the stress ribbon bridge and the chosen pneumatic muscle actuator.

In Section 2.3, it is assumed that the operating point of the linearized system does not change significantly during active vibration control. However, the operating point changes by elastic elongation as a result of uniformly distributed static loads and geometric displacement due to single static loads caused by pedestrians. If only one or two pedestrians walk on the bridge, the static operating point drift can be neglected. But with increasing pedestrian traffic the static operating point varies. Hence, a full state feedback control design using modal "displacement" and "velocity" is unsuitable due to the fact that steady displacement feedback generates a control force trying to counteract static vertical displacement. Consequently, a feedback controller is designed in Section 4.2.2 using only "velocity" feedback.

In principle, energy dissipation can easily be achieved by proportional negative feedback of the velocity because the power is equal to the actuator force multiplied by the velocity [21]. But in the presence of any dynamical behavior in the sensors, actuators, and signal processing used, the control performance can decrease drastically up to instability. As described in Section 3, the actuator force depends on the pressure inside the actuator. To identify the dynamics of pressure build-up, experiments were conducted. In Section 4.2.1, Fig. 8, the results of the experiments show that the dynamics of pressure build-up can be treated as a first-order dynamical system (PT₁-element). Hence, this has to be explicitly considered in the control design. Here, it was done by introducing an additional time delay $T_d$ in the closed loop to bring the force and the velocity in phase in the first mode. This approach enhances the investigations of [22].

4.2.1. Force control

In this section, the actuator dynamics are investigated. To handle the nonlinearities of the PMA as described in Section 3, a subsidiary force control is applied, similar to that proposed in [23]. The force control is based upon the inverse muscle force function.

The input variable is the reference force $F_{ref}$, which is converted to the corresponding reference pressure $p_{ref}$. By using the inverse muscle force function, a static nonlinear gain is calculated:

$$p_{ref}(t) = F_{ref}(s(t), F_{ref}(t)).$$

To identify the pressure characteristics between the reference pressure $p_{ref}$ and actual pressure $p_{act}$, different pressure steps were applied on a hanging muscle and the actual pressure was measured. The behavior resulting from hanging different additional weights on the muscle was also checked. The actual pressure response showed a considerable dependence on the step size and a smaller dependence on the weights carried. For approximation of the nonlinear pressure build-up, a first-order delay transfer function (PT₁) is used:

$$G_p(s) = \frac{1}{T_1 \cdot s + 1}.$$  \hspace{1cm} (35)

The time constant for the transfer function was set to $T_1 = 0.15$ s, according to a 3 bar step, shown in Fig. 8. It should be noted that this approach avoids requiring a detailed description of the nonlinear flow rate and the valve characteristics. However, the pressure response for reference pressures below 3 bar is faster and above 3 bar is slower.

To ensure an asymptotic pressure tracking, a controlled proportional pressure valve was used and the pressure at the valve is measured. Finally, the force control produces a linear input/output behavior approximated by a first-order delay transfer function.

$$F(s) \approx \frac{1}{T_1 \cdot s + 1} F_{ref}(s).$$  \hspace{1cm} (36)

Fig. 8. Pressure response approximated by a first-order dynamical system (PT₁-element).

4.2.2. Delayed modal velocity feedback control design

The input for delayed modal velocity feedback control design is the estimated modal velocity of the first mode $\dot{\mathbf{x}}_{m1}$. The controller design was carried out first in the modal state space, giving a control law for a normalized modal control signal (unit gain between modal velocity and input). Afterwards, this control signal was transformed into the real nodal control signal $F_{ref}$. The transfer function Eq. (38) of the first mode between the modal input and the modal velocity output is derived using the Laplace transform of the modal system matrix $\mathbf{A}_{m1}$ given by Eq. (29):

$$\ddot{\mathbf{x}}_{m1} = \mathbf{A}_{m1} \mathbf{x}_{m1}$$

whereas $\mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$G(s) = \frac{\mathbf{C}}{s^2 + 2\zeta\omega_1s + \omega_1^2}. \hspace{1cm} (37)$$

The transfer function of the additional introduced constant time delay $T_d$ is given by

$$G_d(s) = e^{-sT_d}. \hspace{1cm} (39)$$

Finally, the poles of the closed-loop transfer function of the controlled first mode of the bridge are given by the following equation:

$$1 + G_p(s)G(s)G_d(s)K = 0,$$  \hspace{1cm} (40)

where $G_p(s)$ is the approximated actuator dynamics, $G(s)$ is the first mode of the bridge, $G_d(s)$ is the time delay, and $K$ is the controller gain. Eq. (40) represents a function of $K$ and $T_d$. The first task is to determine the complex conjugated poles $s_{1,2}$ of the controlled system given by Eq. (41). The related real pole $\bar{\zeta}_3$ of the actuator dynamics is less important for velocity feedback control. To get the system poles, Eq. (40) is numerically solved [24] for different
dimensionless time delays $\tau = T_d/T_{n1}$ and different gains $K$, where $T_{n1} = 2\pi/\omega_1 = 0.751$ s is the natural period of the first mode.

$$
\dot{s}_{1,2} = -\frac{\hat{w}_1 \pm i \omega_1}{Re} \sqrt{1 - \hat{\zeta}_1^2}.
$$

Hence, the modal damping ratio $\hat{\zeta}_1$ and the natural angular frequency $\omega_1$ of the controlled first mode can be calculated by

$$
\hat{\zeta}_1 = \frac{Re(s_1)/Im(s_1)}{\sqrt{1 + (Re(s_1)/Im(s_1))^2}}
$$

$$
\hat{\omega}_1 = \frac{Im(s_1)}{\sqrt{1 - \hat{\zeta}_1^2}}.
$$

In Fig. 9, the pole locations in the upper half complex plane are plotted. Each root locus is related to a fixed time delay $\tau$ and variable gains $K = \{0, 0.5, 1, \ldots, 6\}$, starting at the poles of the uncontrolled system $s_{1,2} = -0.018 \pm i 8.36$ when $K = 0$. It can be seen that, with increasing values of time delay $T_d$, the poles cross the unstable right complex plane and move back into the stable left complex plane.

The optimal modal damping ratio $\hat{\zeta}_{1,opt}$ is achieved with an additional time delay $T_d = 0.86 \cdot T_{n1} = 0.646$ s and a gain $K = 2.5$. By this selection the modal velocity and control force are in phase. Up to now, no constraints in the control signal have been taken into account. Due to the nature of the chosen actuator, damping forces can only be applied during the upwards movement of the bridge, as the muscle has to be contracted. To consider this limiting behavior of the actuator, the controller gain $K$ has to be increased by the factor two in order to obtain the poles given by the root locus (Fig. 9).

Finally, the physical control force $F_{ref}$ is given by

$$
F_{ref} = -B_{m1,vel}^{-1} \cdot 2 \cdot K \cdot \dot{s}_{m1}(t - T_d).
$$

where $B_{m1,vel}^{-1}$ is the inverse velocity entry of the modal input vector $B_{m1}$. The results of this control strategy are shown and discussed in Section 5.

4.3. Muscle contraction observer design

One objective always present in the design of active vibration control systems is to use a minimum number of sensors. If possible, an observer should be developed in order to provide an estimate of the system states. To control the muscle force, the muscle contraction length $s$ has to be measured or estimated. From Fig. 4, it can be realized that the distance between the handrail posts, where the PMA is attached, has a geometric relation with the vertical displacement of the bridge. An approximated geometric relation, Eq. (38), can be expressed by using a two-plate model, where $w_1 = L/4$. For further parameters, see Table 1.

$$
s_{est}(x_T) = -\frac{w_1 x_T^2}{(L_T/2)^2} + \frac{2(h + 1/2)d x_T}{L_T/2}.
$$

5. Simulations and experiments

5.1. Details for simulation and implementation of the control design

First, the simulations are carried out using the open-source software package Scilab/Scicos for numerical computation (http://www.scilab.org/, http://www.scicos.org/). The controller gain for delayed velocity feedback control was determined and tested by using the eight-plate model in the simulation environment, as shown in Fig. 10. The excitation signal for the simulations is a sinusoidal pressure signal in the first mode for an initial 4 s. The pneumatic muscle actuators work first as an exciter, and they cause a maximum displacement amplitude of $\pm 2.5$ cm in the midspan. Afterwards, the vibrations are actively damped by the actuator. This kind of excitation was chosen to make the comparison of the results of the simulation and experiment well defined, because there are too many differences between actual pedestrian excitations and approximated pedestrian forces for the simulations. To consider sensor and actuator noise, the estimated force $F_{est}$ and the output signals $y$ are superimposed by noise.

In a next step, the control structure was implemented in a real-time environment with a sampling frequency of 100 Hz to
carry out experiments on the real bridge. Therefore, a PC running a Linux operation system with the real-time extension RTAI (https://www.rtai.org/) is used. The software code was generated and compiled from Scilab/Scicos block diagrams. Communication to devices was provided through the HART Toolbox (http://hart.sourceforge.net/), [25]. For the experiment, the controlled system in Fig. 10 is replaced by the bridge. The vertical displacement in the midspan is measured by a displacement encoder MLO POT-360-LWG [18], whereas the velocity is obtained from numerical differentiation of the displacement. Low-pass filters are used to clean the signals $y$ from noise. To validate the control performance by means of estimated muscle contraction $s_{est}$ and estimated muscle force $F_{est}$, a displacement encoder and a load cell are placed between the handrail posts, as shown in Fig. 3b. This allows two options to control the actuator: first, by measured muscle contraction and force or second, by estimated muscle contraction and estimated force using the muscle force function (Eq. (27)). The reference pressure $p_{ref}$ is in the range of 0–6 bar. Each PMA is driven by a single proportional valve, type VPPM-6F-L-1-F-0L6H-A4P-S1 [18].

5.2. Results

The results of simulation and experiment with delayed velocity feedback control after excitation by the actuator are shown in Fig. 11. In the upper diagram, the estimated (est) displacement and velocity signals of the simulation (sim) and of the experiment (exp) are shown. In the lower diagram, the reference (ref) force $F_{ref}$ and the measured (act) force for the simulation and the experiment are plotted. It can be seen that the results of the simulation and
the experiment are in good agreement. The assumption of a first-order delay system for pressure build-up is suitable for pressure steps of about 3 bar, as already mentioned in Section 4.2.1. Lower or higher pressure steps cause small discrepancies between $P_{ref}$ of the simulation and the experiment. The control performance using estimated muscle contraction/force is similar to the performance using measured data.

In Fig. 12, time histories of the estimated modal velocity with delayed velocity feedback control and without control are plotted. The control strategy efficiently reduces the first mode response. The uncontrolled response decays very slowly, with logarithmic decrement $\Lambda_{1,\text{uncont}} \approx 0.034$. The controlled response decays about 20 times more quickly, with $\Lambda_{1,\text{cont}} \approx 0.812$. The controller gain was set to not exceed the maximum possible force $F_{\text{max}} = 3000$ N of one PMA.

The control performance under pedestrian excitation is shown in Fig. 13. One person is walking from one side to the other side with a step frequency of 1.35 Hz that corresponds to the bridge’s first natural frequency. Without control, a maximum acceleration amplitude of 6 m/s² occurs. With control, the acceleration amplitude can be reduced by $\approx 70\%$, to less than 2 m/s².

In the present control system, only the first mode is controlled. As described in Section 4.1, the measurement of the vertical displacement $x_7$ at the midspan allows the estimation of four symmetrical modes. Thereby, the instability of higher modes due to spillover effects could be observed. In Fig. 14 it is shown that there are no significant spillover effects in higher symmetrical modes. Spillover effects in higher unsymmetrical modes could not be observed.

6. Conclusion

Active vibration control for a stress ribbon bridge with an extremely light pneumatic actuator was investigated. Therefore, an analytical nonlinear bridge model was developed. By experiments,
it was confirmed that the linearized model represents the nonlinear behavior of the stress ribbon well for multi-modal motions. By using sensors to measure the nodal displacement and pressure only, a subsidiary force control was presented which compensates the nonlinearity of the pneumatic actuator. Therefore, the actuator dynamics was considered and approximated by a first-order delay system. The proposed overlaying delayed velocity feedback control strategy to actively damp the bridge mitigates the first mode responses efficiently. To verify the model-based control designs, full-scale experiments were conducted.

It could be observed that the control performance mainly depends on the delayed nonlinear pressure build-up. To improve the control performance, faster proportional pressure valves with a faster pressure control should be implemented especially to control higher modes. To match the pressure build-up in simulation with the real pressure characteristics, a detailed model of the dynamic pressure build-up would be necessary.

This control strategy can be easily extended to multi-variable control; thus in further studies a multi-variable control system will be implemented to control asymmetric and higher symmetric modes. In addition, the efficiency of the controller in reducing forced and random vibrations induced by a number of pedestrians will be investigated [4].

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References

[16] Lunze J. Regelungstechnik 2; Regelungstechnik 2. revised ed. Berlin: Springer Verlag; 2007. [in German].