Two-Stage Consensus-Based Distributed MPC for Interconnected Microgrids

A. K. Sampathirao, C. A. Hans, and J. Raisch

Abstract—In this paper, we propose a model predictive control based two-stage energy management system that aims at increasing the renewable infused in interconnected microgrids (MGs). In particular, the proposed approach ensures that each MG in the network benefits from power exchange. In the first stage, the optimal islanded operational cost of each MG is obtained. In the second stage, the power exchange is determined such that the operational cost of each MG is below the optimal islanded cost from the first stage. In this stage, a distributed augmented Lagrangian method is used to solve the optimisation problem and determine the power flow of the network without requiring a central entity. This algorithm has faster convergence and same computational requirements. The properties of the algorithm are illustrated in a numerical case study.

Index Terms—Operation of interconnected microgrids, model predictive control, distributed augmented Lagrangian, dual decomposition.

I. INTRODUCTION

Recent advancements in renewable energies and growing concerns about environmental impacts from fossil fuelled power stations lead to worldwide increase in installation of renewable energy sources (RES) such as photovoltaic plants and wind turbines [1]. RES are often small-scaled distributed units (DUs), characterised by an intermittent power output and by geographical proximity to consumers. At this juncture, the microgrid (MG) concept is a promising direction to facilitate the integration of a large number of RES. An MG refers to a self-contained system with local demand, generation and storage units. MGs can operate either connected to or islanded from the power network [2].

Connecting geographically close MGs and facilitating power exchange between these MGs can increase the flexibility in their operation [3]. This offers new economic markets for MGs as they can benefit from the technological and temporal diversity of the different RES. The objective of this paper is to develop a distributed control strategy for power exchange preserving the self-interests of the MGs in a network. The resulting control scheme has two-stages. The first stage estimates the optimal islanded cost. The second stage considers power exchange between the MGs in the network such that each MG benefits from it compared to the islanded case. The underlining scheme at each layer is model predictive control (MPC). MPC is widely adopted for the energy management of MG as it allows to explicitly consider operational limits of the units and transmission lines in the MG [4], [5].

Coordinating multiple MGs in a network has been already discussed in literature, e.g., in [5], [6], [7] and the references therein. Ouammi et al. [5] proposed an MPC based central controller to determine the power from the DUs in each MG and manage the power flows in the network. Wang et al. [6] adopted a hierarchical architecture with a network controller to manage the transmission grid and the local controllers in each MG to schedule the generation of the units. A similar architecture has been discussed in [8] for the interaction between the main grid and an MG cluster. All of the aforementioned publications highlight the benefits of coordinated power transfers in interconnected MGs. However, they all assume a centralised architecture. Furthermore, they focus on minimising the combined operating costs of all MGs. Therefore, these approaches are not suitable for instances where the MGs are operated by different entities. First, minimising the global cost does not secure the self-interests of every individual MG. Second, the centralised architecture can undermine the privacy of independent MGs.

The first issue was partly tackled by Parisio et al. [9] by including shared resources, e.g., combined heat and power plants. In [9], a scheme was presented where first local optimisations are performed in parallel by all MGs. In a second phase, shared resources are added that allow to further decrease the cost for every individual MG. Still, the MGs can only benefit from an interconnection because of the added resources assumed in the second phase. Furthermore, the approach in [9] requires a central coordinator.

Recently, approaches based on distributed algorithms like the alternating direction method of multipliers (ADMM), or dual decomposition (DD) were proposed to address the disadvantages of a centralised architecture (see, e.g., [10], [11]). A main limitation of [10], [11] is their exclusion of the on/off state of conventional generators. This is due to the lack of effective methods to include integer variables in distributed formulations. Hans et al. [12] proposed a hierarchical distributed MPC that includes the switching of conventional generators. The binary variables that represent the on/off state of the conventional generators were relaxed and the power transfer was decided in a distributed manner using the ADMM. However, the MGs have to be designed in a certain way to
ensure feasibility of the proposed algorithm. Furthermore, the approach assumes a central entity that communicates with all MGs which makes the control scheme prone to single-point failures.

The main contributions of this paper are as follows.

1) We propose an MPC based two-stage control scheme that includes trading of energy in a way that allows every MG in the network to benefit from the exchange. In the first stage, optimal cost for each islanded MG is obtained. In the second stage, power exchange is found such that the operational cost of all MGs is below the islanded cost.

2) A feasible sub-optimal formulation of the mixed integer problem is solved to determine the power flow in the network. Here the integer variables are fixed to the output of the first stage. The second stage problem is convex and suitably reformulated with consensus constraints to solve in distributed manner without a central coordinating entity. Distributed augmented Lagrangian (AL) method is employed to solve this MPC problem.

3) Each iteration of the proposed AL algorithm requires the same information exchange as DD [3]. However, the algorithm is numerically robust and known to converge faster than DD as demonstrated in a case study.

The remainder of the paper is structured as follows. In Section II we discuss the MG and network model. Section III describes the operational costs and formulates the two-stage MPC problem solved to control the power exchange. Then, in Section IV the second stage problem is stated and reformulated using consensus constraints that are used for distributed implementation. Section V presents a numerical case study that illustrates the performance of the distributed AL algorithm along with benefits of trading power between the MGs.

A. Notation and mathematical preliminaries

The set of real numbers is denoted by $\mathbb{R}$, the set of nonpositive real numbers by $\mathbb{R}_{\leq 0}$ and the set of positive real numbers by $\mathbb{R}_{>0}$. The set of extended-real numbers is denoted by $\mathbb{R} = \mathbb{R} \cup \{\pm \infty\}$. The set of positive integers is denoted by $\mathbb{N}$ and the set of the first $n$ positive integers by $\mathbb{N}_n = \{1, 2, \ldots, n\}$. The cardinality of a set $A$ is $|A|$. The vector $[x_{v_1}, x_{v_2}, \ldots, x_{v_n}]$ for all $v_i \in V = \{v_1, v_2, \ldots, v_n\} \subset \mathbb{N}$ with $v_i < v_j$ for $i < j$ is denoted by $\text{vec}(x_{v_i})_{v_i \in V}$. We denote $\text{vec}(x_{v_i,u_j})_{v_i \in V, u_j \in U}$ for the vector $\text{vec}(\text{vec}(x_{v_i,u_j})_{u_j \in U})_{v_i \in V}$.

The indicator function of a set $C \subseteq \mathbb{R}^n$ is the extended real valued function $I_C : \mathbb{R}^n \rightarrow \mathbb{R}$. For $x \in C$, $I_C(x) = 0$ and for $x \not\in C$, $I_C(x) = +\infty$. Furthermore, $\|\cdot\|$ is the $L_2$ norm and $|\cdot|$ is the absolute value.

An undirected weighted connected graph $G$ is an ordered triple $G = (N, E, y)$ where $N = \{1, 2, \ldots, n\}$ is the set of nodes, $E$ is the set of edges linking the nodes and $y \in \mathbb{R}^{|E|}$ is a vector of weights associated with the edges of the graph. The set $E \subseteq |E|^2$, where $|E|^2$ denotes the set of all subsets of $E$ with elements $(i, j), \forall i, j \in N$. Node $j$ is a neighbour of node $i$, if there is an edge $(i, j) \in E$. At node $i$, the set of neighbours is defined as $N_i = \{j \mid j \in N, (i, j) \in E\}$. Let us denote the number of neighbours of node $i$ as $n_i = |N_i|$.

II. System Description and Modelling

In this section we present the discrete-time model of a network of MGs that will serve as a basis for the control design. We first present the model of a single MG. Then, the model of the electrical grid connecting the MGs is introduced. We start by introducing some modelling basics.

The model is motivated by [12] and considers $n$ MGs that are connected by an electrical network as shown in Figure 1. Each MG is denoted by MG$_j$ with $j \in \{1, 2, \ldots, n\}$ and connected to a bus in the electric grid by a point of common coupling (PCC) that allows for power exchange with other MGs. Still, each MG is assumed to be capable of operating in islanded mode with zero power exchange at the PCC.

A. Model of single microgrid

An MG is an electrical system composed of DUs and loads. We assume that all DUs in an MG are either conventional generators, storage devices or RES. Let us denote the number of DUs and loads by $g_j$ and $l_j$ for MG$_j$. Each unit is labelled as DU$_{j,i}$ with $i \in \{1, 2, \ldots, g_j\}$. Furthermore, we define the index set of the conventional generators with $C_j = \{i \mid i \in \mathbb{N}_{g_j}, DU_{j,i}$ is a conventional generator$\}$. Similarly, we define $S_j$ as the index set of storage units and $R_j$ as the index set of RES. In the same way, the loads are labelled as DL$_{j,i}, i \in \{1, 2, \ldots, l_j\}$ and the index set of the loads as $L_j = \{1, 2, \ldots, l_j\}$. Note that the number of conventional generators, storage units and renewable units in MG$_j$ is $|C_j|$, $|S_j|$ and $|R_j|$ with $g_j = |C_j| + |S_j| + |R_j|$.

1) Renewable energy sources: RES like photovoltaic plants or wind turbines are assumed to be the predominant energy providers in the MGs. The power from these units is uncertain as their availability depends on intermittent weather conditions, e.g., solar irradiation or wind speed. At time instance $k \in \mathbb{N}$, the available power from RES is denoted by $w_{j,r}(k)$. The actual power indeed of a renewable unit $p_{j,r}(k)$ is limited by the available power $w_{j,r}(k)$ and the maximum power, $p_{j,r} \in \mathbb{R}_{\geq 0}$, i.e.,

$$0 \leq p_{j,r}(k) \leq \min(p_{j,r}, w_{j,r}(k)), \quad \forall r \in R_j. \quad (1)$$
2) Storage units: We consider battery-based storage devices to compensate the fluctuations from the RES by charging in times of high RES infeed and discharging in times of low RES infeed. The energy stored and the power is denoted by \( x_{j,s}(k) \) and \( p_{j,s}(k) \), \( s \in S_j \). The bounds on the stored energy and the power are
\[
\begin{align*}
\delta x_{j,s} \leq x_{j,s}(k) &\leq \bar{x}_{j,s}, \quad \forall s \in S_j, \\
\underline{p}_{j,s} \leq p_{j,s}(k) &\leq \bar{p}_{j,s}, \quad \forall s \in S_j,
\end{align*}
\]
with \( \delta x_{j,s} \in \mathbb{R}_{\geq 0}, \bar{x}_{j,s} \in \mathbb{R}_{\geq 0}, \underline{p}_{j,s}, \bar{p}_{j,s} \in \mathbb{R} \) and \( \overline{p}_{j,s} \in \mathbb{R} \). For a given power \( p_{j,s}(k) \), the dynamics of storage unit \( s \) is
\[
x_{j,s}(k + 1) = x_{j,s}(k) - T_s p_{j,s}(k), \quad \forall s \in S_j,
\]
where \( T_s \in \mathbb{R}_{\geq 0} \) is the sampling time. If \( p_{j,s} < 0 \), the battery is charging, i.e., acting as a load. If \( p_{j,s} > 0 \), the battery is discharging, i.e., acting as a generator.

3) Conventional generators: Typically, conventional generators are used as backup in times of low renewable infeed and low storage energy. Let us represent the switch state of unit \( c \in C \) with the binary variable \( \delta_{j,c} \in [0,1] \) where \( \delta_{j,c} = 0 \) means that the generator is disabled and \( \delta_{j,c} = 1 \) that it is enabled. We assume that the time required to enable or disable the DU is much smaller than the sampling time of the energy management system (EMS) in the MG. Therefore, switching is assumed to be controllable.

The power output of a conventional generator is limited by
\[
p_{j,c}(k) \delta_{j,c} \leq p_{j,c}(k) \leq \bar{p}_{j,c} \delta_{j,c}, \quad \forall c \in C_j,
\]
with limits \( \underline{p}_{j,c}, \bar{p}_{j,c} \in \mathbb{R}_{\geq 0} \) and \( \overline{p}_{j,c} \in \mathbb{R}_{\geq 0} \). Note that the above constraint is equivalent to \( p_{j,c}(k) = 0 \) for \( \delta_{j,c}(k) = 0 \) and in \( p_{j,c}(k) \leq \overline{p}_{j,c} \delta_{j,c}(k) = 1 \).

4) Loads: The loads in the network represent the electric demand in the MGs. We assume that the demand cannot be controlled, i.e., it is part of the uncertain disturbance inputs.

At instance \( k \), the power of load \( l \in L_j \) is \( w_{j,l}(k) \in \mathbb{R}_{\leq 0} \). At instance \( k \), the power of load \( l \in L_j \) is \( w_{j,l}(k) \in \mathbb{R}_{\leq 0} \). At instance \( k \), the power of load \( l \in L_j \) is \( w_{j,l}(k) \in \mathbb{R}_{\leq 0} \).

5) Point of common coupling (PCC): The PCC connects the MG with the transmission network and thus allows for power exchange with the network. Let us denote the power transferred to the network by \( p_{j,p}(k) \in \mathbb{R} \). If \( p_{j,p}(k) < 0 \), power is provided by the MG and if \( p_{j,p}(k) > 0 \) power is consumed by the MG. The power at the PCC depends on the power of the DU and loads in MG, it can be determined using the power balance equation
\[
p_{j,p}(k) + \sum_{i=1}^{g_s} p_{j,i}(k) + \sum_{i=1}^{l_j} w_{j,i}(k) = 0.
\]

B. **Electrical grid model**

The electrical grid, i.e., the network of power lines that connect the individual MGs is modelled as an undirected graph \( G = (N,E,y) \) (see Section I-A). Every MG connected to the grid is represented by one node in the set \( N = \{1,2,\ldots,n\} \). Every transmission line in the grid is represented by one edge in \( E \). The weights in the graph, \( y_{jm} \in \{j,m\} \in E \) correspond to the susceptances of the electrical lines in the network. We assume constant voltage magnitude at all the nodes, lossless power lines and small phase angle differences [12]. Thus, we can use the DC power flow approximations to model the line power in the grid.

Let us denote the phase angle at node, \( j \), by \( \theta_{j}, j \in N \). The power of line connecting nodes \( (MGs) \ j \) and \( m \in N \) is denoted as \( p_{jm}(k) \). This power is limited by a maximum and minimum line power \( \chi_{jm} \in \mathbb{R} \) and \( \chi_{jm} \in \mathbb{R} \).

\[
p_{jm}(k) \leq p_{jm}(k) \leq \overline{p}_{jm}, \forall s \in S_j, \quad j, m \in E.
\]

The power provided by node \( j \) via the PCC can be described by the sum of the power of all lines connected to it, i.e.,
\[
p_{j,p}(k) = \sum_{m \in N_{j}} y_{jm}(\theta_{j,j}(k) - \theta_{j,m}(k)),
\]
(see, e.g., [13])

\[
p_{j,m}(k) = y_{jm}(\theta_{j,j}(k) - \theta_{j,m}(k)), \quad \forall j \in N.
\]

Remark 2.1: As the lines are assumed to be lossless, a global power balance holds, i.e., \( \sum_{j \in N} p_{j,p}(k) = 0 \), where \( p_{j,p}(k) \) is the power that MG exchanges with the other MGs. However, this condition is already implicitly included in (5b) and (5c) (see, e.g., [12]).

Combining the grid model, the constraints and dynamics, i.e., [11–13], results in a mathematical model. This model can now be used in the control design in the next section.

III. **MPC based EMS for interconnected MGs**

The objective of the EMS for interconnected MGs is, amongst others, to increase the overall renewable infeed and reduce conventional generation. In this section, we formulate a cost function that reflects these objectives. Furthermore, we pose a two-stage MPC based EMS that ensures that every MG in the network benefits from exchanging power.

A. **Control objectives**

The control objective comprises the operational cost of each MG and of the electrical network. The cost of MG \( j \) is
\[
\ell_{j}(z_{j},\delta_{j}) = \sum_{i \in C_{j}} \ell_{j,c}(p_{j,i},\delta_{j,i}) + \sum_{i \in R_{j}} \ell_{j,r}(p_{j,i}) + \sum_{i \in E_{j}} \ell_{j,s}(p_{j,i}) + \ell_{j,p}(p_{j,j}),
\]
(6a)

where \( z_{j} = [p_{j,1} p_{j,2} \cdots p_{j,q_j} p_{j,a}] \) and \( \delta_{j,i} = \text{vec}(\delta_{j,c})_{c \in C_{j}} \). The operating cost for the conventional, storage and renewable units is given by \( \ell_{j,c}(\cdot,\cdot) \in \mathbb{R}_{\geq 0}, \ell_{j,s}(\cdot) \in \mathbb{R}_{\geq 0} \) and \( \ell_{j,r}(\cdot) \in \mathbb{R}_{\geq 0} \). The term \( \ell_{j,p}(\cdot) \in \mathbb{R} \) presents the cost associated with the exchange of power via the PCC.

We assume that RES do not have any cost when operated and that it is desirable to maximise their infeed. Therefore the cost \( \ell_{j,r}(\cdot) \) aims at penalising the limitation of renewable infeed. This objective can be expressed by choosing
\[
\ell_{j,r}(p_{j,r}) = a_{j,r}(p_{j,r} - w_{j,r})^{2},
\]
(6b)

where \( a_{j,r} \in \mathbb{R}_{>0} \) is a weight and \( w_{j,r} \) is the available renewable power of unit \( r \in R_{j} \).

For the storage devices, no storage losses and no costs that depend on the stored energy were assumed. Still, charging or...
discharging at high power can have a negative effect on the ageing of the batteries [15]. This is included in the operational cost of storage unit \( s \in S_j \) by
\[
\ell_{j,s}(p_{j,s}) = a_{j,s}p_{j,s}^2,
\]
where \( a_{j,s} \in \mathbb{R}_{>0} \) is a weight.

The operational cost of conventional generator \( \ell_{j,c}(\cdot, \cdot) \) consider the fuel costs. This can be approximated for all \( c \in C_j \) by the quadratic function (see, e.g., [15])
\[
\ell_{j,c}(p_{j,c}, \delta_{j,c}) = a_{j,c}\delta_{j,c} + a'_{j,c}(p_{j,c}) + a''_{j,c}p_{j,c}^2,
\]
with weights \( a_{j,c} \in \mathbb{R}_{>0}, a'_{j,c} \in \mathbb{R}_{>0} \) and \( a''_{j,c} \in \mathbb{R}_{>0} \).

Finally, the power exchange with the electric grid via the PCC is included in the cost function by
\[
\ell_{j,p}(p_{j,p}) = a_{j,p}p_{j,p} + a'_{j,p}p_{j,p}^2,
\]
where \( a_{j,p} \in \mathbb{R}_{>0} \) represents the selling/buying price of the power and \( a'_{j,p} \in \mathbb{R}_{>0} \) additional costs for trading.

A cost that is not included in (6a) is associated with the transmission network. We assume the cost of transferring power in a line can be approximated by the quadratic function
\[
\ell_{l}(z_l) = \sum_{j \in N} \sum_{m \in N_j} a_{jm}z_{jm}^2,
\]
with \( z_l = \text{vec}(z_{jm})_{j \in N, m \in N_j} \) and \( a_{jm} = a_{m,j} \in \mathbb{R}_{>0} \).

Using \( z(k) = [z_1(k) \cdots z_n(k)] \) and \( \delta(k) = [\delta_1(k) \cdots \delta_n(k)] \), the stage cost at time instance \( k \) can be expressed as the sum of the above costs, i.e.,
\[
\ell(z(k), \delta(k)) = \sum_{j=1}^n \ell_j(z_j(k), \delta_j(k)) + \ell_l(z_l(k)).
\]

### B. Central two-stage MPC

Even though the MGs in the interconnected network can exchange power, they are still independent and each of them prefers minimising its own operating cost. Most control schemes proposed for MG networks find the optimal operation for the whole network, often at the expense of single MGs. We propose an alternative control strategy that ensures that every MG benefits from power trading. This is realised using a two-stage control scheme. In the first stage, the optimal islanded cost of each MG is calculated. In the second stage, the MGs are considered to be in grid-connected operation. Now, the cost from the first stage is used as an upper bound on the cost of every MG to ensure that all MGs benefit from trading.

The control schemes of both stages are realised using MPC. MPC is an optimal control based approach that finds the input trajectories by minimising the operational cost over a finite horizon subject to constraints [17]. In MPC, only the first value of the predicted trajectories are applied at each sampling time and the rest of the input sequence is discarded. At the next sampling time instance the problem is updated with new measurements, predictions and the procedure is repeated.

1) **Stage I (Islanded MGs):** The first stage of the control scheme obtains the optimal islanded operation cost for each MG. This cost is calculated by solving an MPC problem. This is formulated using the model presented in Section III and the costs from Section III-A.

The decision variables at each MG are the power of all units and the switch state of the conventional generators. At MG\(_j\), let us define the decision variables over the prediction horizon \( H \in \mathbb{N} \) as \( z_j = [z_{j}(k) \cdots z_{j}(k+H[k])] \) and \( \delta_j = [\delta_{j}(k) \cdots \delta_{j}(k+H[k])] \). Here \( z_{j}(k+h) \) denotes the prediction for step \( h \in \mathbb{N}_H \) (i.e., for time instance \( k+h \)), performed at time instance \( k \).

The forecasts of the renewable infeed and the load demand are represented by
\[
\hat{w}_j(k+h|k) = [\text{vec}(\hat{w}_{j,s}(k+h|k))_{r \in R_j}, \text{vec}(\hat{w}_{j,l}(k+h|k))_{l \in L_j}],
\]
for all \( j \in N \). With this forecast, we can formulate the set of feasible power values for MG\(_j\) as
\[
Z_j(\hat{w}_j, \delta_j) = \left\{ z_j \in \mathbb{R}^{g_{j,H+1}} | \sum_{h=0}^{H} p_{j,p}^h \leq p_{j,s} \leq \bar{p}_{j,s}, \forall s \in S_j, p_{j,c} \delta_{j,c} \leq p_{j,c} \leq p_{j,c} \delta_{j,c}, \forall c \in C_j, 0 \leq p_{j,r} \leq \min(p_{j,r}, \hat{w}_{j,r}), \forall r \in R_j, p_{j,p} + \sum_{l=1}^{L_j} \hat{w}_{j,l} = 0 \right\}.
\]

Let us further denote the energy and the power of all the storage devices in MG\(_j\), by \( x_j = \text{vec}(x_{j,s})_{s \in S_j} \) and \( z_{j,s} = \text{vec}(z_{j,s})_{s \in S_j} \). Then, the energy constraints of the storage devices, (2a), can be captured by the set
\[
X_j = \{ x_j \in \mathbb{R}^{S_j} | x_{j,s} \leq \bar{x}_{j,s} \leq \bar{x}_{j,s}, \forall s \in S_j \}.
\]

Using the initial measurement of the energy in the storage devices \( x_0 \) in MG\(_j\) and the discount factor \( \gamma \in (0,1] \), the MPC problem of islanded MG\(_j\), over prediction horizon \( H \) can now be formulated as follows.

**Problem 1 (Islanded MPC):**

\[
\min_{z_j, \delta_j} V_{j}(z_j, \delta_j)
\]
subject to
\[
\begin{align*}
V_{j}(z_j, \delta_j) &= \sum_{h=0}^{H} \gamma^h f_j(z_j(k+h|k), \delta_j(k+h|k)) \\
x_j(k|k) &= x_0(j|k), \quad (1a) \\
x_j(k+h+1|k) &= x_j(k+h|k) - T_{s} z_{j,s}(k+h|k), \quad (1b) \\
x_j(k+h+1|k) &\in X_j, \quad \delta_j(k+h|k) \in \{0,1\}^{C_j}, \quad (1c) \\
z_j(k+h|k) &\in Z_j(\hat{w}_j(k+h|k), \delta_j(k+h|k)), \quad (1d) \\
p_{j,p}(k+h|k) &= 0, \quad (1e)
\end{align*}
\]
for all \( h \in \mathbb{N}_H \).

Let us denote the switch state and power obtained by solving Problem 1 by \( \delta_j^{*\text{I}} = [\delta_j^{*\text{I}}(k) \cdots \delta_j^{*\text{I}}(k+H[k])] \) and \( z_j^{*\text{I}} = [z_j^{*\text{I}}(k) \cdots z_j^{*\text{I}}(k+H[k])] \), where \( I \) in the superscript is used to mark the islanded case. Then, the optimal islanded operational cost is \( V_{j}(z_j^{*\text{I}}, \delta_j^{*\text{I}}) \). Note that in Problem 1 \( p_{j,p}(k+h|k) = 0 \) ensures an islanded operation as the PCC power is forced to zero for all MG\(_j\), \( j \in N \).
2) Stage II (Interconnected MGs): The second stage of the control scheme decides the power exchanges between the interconnected MGs. In this stage, the MPC problem is formulated for the entire system, the MGs and the network.

Let us denote the power at PCCs in the interconnected network by \( z_p = \text{vec}(p_{j,p})_{j \in N} \). Then, the network operational constraints can be formulated with the set

\[
\mathcal{Z}(z_p) = \{ p_{jm} \in \mathbb{R}, \forall (j,m) \in E \mid p_{jm} \leq p_{jm} \leq \mathcal{P}_{jm} \},
\]

\[
p_{jm} = y_{jm} (\theta_{jm} - \theta_{mm}), \quad p_{j,p} = \sum_{m \in N_j} p_{jm} \}.
\]

Along with this, each MG’s operational cost is bounded by the islanded operational cost of the first stage. The corresponding feasible set of power values at each MG is

\[
\mathcal{C}_j(\delta_j, z_j^{*I}, \delta_j^{*I}) = \{ z_j \in \mathbb{R}^{H_j+1} \mid V_j(z_j, \delta_j) \leq V_j(z_j^{*I}, \delta_j^{*I}) \}.
\]

Let us define the decision variables for a prediction horizon \( H \) as \( Z = [z(k|k) \cdots z(k+H|k)] \) and \( \delta = [\delta(k|k) \cdots \delta(k+H|k)] \). The MPC for the interconnected system can then be formulated as follows.

**Problem 2 (Central interconnected MPC):**

\[
\min_{Z,\delta} \sum_{h=0}^{H} \gamma^h \ell_h(z_h(k+h|k)) + \sum_{j=1}^{n} V_j(z_j, \delta_j)
\]

subject to

\[
x_j(k|k) = x_j^0, \quad x_j(k+h+1|k) = x_j(k+h|k) - T_s z_{j,s}(k+h|k),
\]

\[
x_j(k+h+1|k) \in \mathcal{X}_j, \quad \delta_j(k+h|k) \in \{0,1\}^{|\mathcal{G}_j|},
\]

\[
z_j(k+h|k) \in \mathcal{Z}_j(\bar{v}_j(k+h|k), \delta_j(k+h|k)),
\]

\[
z_j(k+h|k) \in \mathcal{Z}_j(\bar{z}_p(k+h|k)),
\]

\[
z_j \in C_j(\delta_j, z_j^{*I}, \delta_j^{*I}),
\]

for all \( h \in \mathbb{N}_H \) and \( j \in N \).

Note that \( \mathcal{I}_j \) is an upper bound on the cost of MG\( j \). However, Problem 2 is nonconvex due to the binary variables. Furthermore, it is a centralised formulation, i.e., the problem is solved by one entity. One approach for distributed computation is decomposing this problem into \( n+1 \) subproblems, \( n \) MGs and the electrical network that coordinates the power flow (see, e.g., \([7], [9]\)).

\[\text{To ensure convergence of the distributed algorithm, the above problem needs to be convexified, e.g., by relaxing the binary variables as in [12].}\]

The drawback of this formulation is that the coordinating controller is prone to single-point failures. Besides, relaxing the binary variables does not guarantee a feasible solution to the original problem. Therefore, in the next section we will propose a convexification that ensures feasibility and solves the convexified problem with a distributed approach that does not require a central coordinator.

**IV. FEASIBLE DISTRIBUTED TWO-STAGE MPC**

In this section we will first present a feasible convex formulation of Problem 2. Then, we will reformulate this problem using additional consensus constraints. Later this problem will be solved using a distributed AL algorithm.

**A. Central two-stage MPC with fixed binary variables**

As in the last section, the first stage of the control scheme solves Problem 1, i.e., the islanded MPC. In the second stage of the proposed formulation the switch states \( \delta \) are fixed to the solution of the first stage. A modified MPC problem that only considers \( Z \) as decision variables is then solved. The modified problem for the interconnected system is as follows.

**Problem 3 (Central convex interconnected MPC):**

\[
\min_{Z} \sum_{h=0}^{H} \gamma^h \ell_h(z_h(k+h|k)) + \sum_{j=1}^{n} V_j(z_j, \delta_j^{*I})
\]

subject to

\[
x_j(k|k) = x_j^0, \quad x_j(k+h+1|k) = x_j(k+h|k) - T_s z_{j,s}(k+h|k),
\]

\[
x_j(k+h+1|k) \in \mathcal{X}_j, \quad \delta_j(k+h|k) \in \{0,1\}^{|\mathcal{G}_j|},
\]

\[
z_j(k+h|k) \in Z_j(\bar{v}_j(k+h|k), \delta_j^{*I}(k+h|k)),
\]

\[
z_j(k+h|k) \in Z_j(\bar{z}_p(k+h|k)),
\]

\[
z_j \in C_j(\delta_j^{*I}, z_j^{*I}, \delta_j^{*I}),
\]

for all \( h \in \mathbb{N}_H \) and \( j \in N \).

**Remark 4.1:** As the binary variables are fixed to the switch states from Problem 1, the modified problem is convex. Also, the power outputs of the islanded MGs are always a feasible solution to the above problem. Therefore, the modified problem is convex and guaranteed to have a feasible solution. This would not necessarily be the case with relaxed binary variables, i.e., with \( \delta_j(k+h|k) \in [0,1]^{|\mathcal{G}_j|} \).

**Remark 4.2:** The decision space of the modified problem is a subset of the original Problem 1. The optimal value of cost function obtained from Problem 3 is always less than or equal to the optimal value obtained from Problem 1. Thereby, the modified problem would result in power setpoints that are sub-optimal compared to the Problem 1.

As Problem 3 is a convex problem, it can be decomposed and solved using distributed optimisation strategies. This will be illustrated in the following sections.

**B. Consensus-based distributed MPC**

The basic idea of the consensus-based distributed MPC approach is that each MG communicates with the neighbouring MGs and reaches consensus regarding the phase angles at the PCC of the MGs. The power at the PCC can be expressed as a function of the phase angles using (5c).

In distributed MPC, at MG\( j \) the local variables are introduced to replicate the phase angle of the adjacent MGs. Let us denote these additional variables by \( \theta_{jm}, m \in N_j \). Further let us collect all the phase angles in \( \theta_j = [\theta_{j,j}, \theta_{j,c}] \) where \( \theta_{j,c} = \text{vec}(\theta_{jm})_{m \in N_j} \) is the vector of replicated phase angles. Let us also define the vector with the actual neighbouring phase angles of MG\( j \), as \( \bar{\theta}_{j,j} = \text{vec}(\theta_{mm})_{m \in N_j} \). The duplicated phase angles are coupled to the phase angles of the neighbours with the constraint

\[
\theta_{jm} = \theta_{mm}, \forall m \in N_j, j \in N.
\]

This is equivalent to \( \theta_{j,c} = \bar{\theta}_{j,j}, j \in N \).
For each transmission line, the constraints related to the line power and the PCC bus given by (5) can be rewritten with the local variables at MG$_j$ for all $m \in N_j$, as

\begin{align}
p_{jm} &= y_{jm}(\theta_{jj} - \theta_{jm}), \\
p_{jp} &= \sum_{m \in N_j} y_{jm}(\theta_{jj} - \theta_{jm}), \\
p_{jm} &\leq p_{jm} \leq \overline{p}_{jm}.
\end{align}

Thus, the constraint set for the phase angles of MG$_j$ is

\[ \Theta_j(p_{jp}) = \{ \theta_{j} \in \mathbb{R}^{|N_j|+1} \mid (10) \}. \]

The transmission cost associated with the lines connected to MG$_j$ in (6f) can be rewritten as

\[ f_j(x_j, \theta_j) = \sum_{m \in N_j} a_{jm}(y_{jm}(\theta_{jj} - \theta_{jm}))^2. \]

Let us denote the decision variables related to the phase angles by $\theta = [\theta_1 \cdots \theta_j]$ and $\theta_j = [\theta_j(k) \cdots \theta_j(k+H[k])]$. Similarly, we can define the vectors $\theta_{jd}, \theta_{jd, c}$ and $\theta_{jd,j}$. With them, the MPC Problem 3 can be rewritten as follows.

**Problem 4 (Distributed convex interconnected MPC):**

\[ \min_{Z, \theta} \sum_{j \in \mathcal{N}} f_j(z_j, \theta_j) \]

subject to

\begin{align}
x_j(k) &= x_j^0, \\
x_j(k+h+1) &= x_j(k+h) - T_k z_{j,x}(k+h), \\
\theta_{jm}(k+h) &= \theta_{mm}(k+h), \quad \forall m \in N_j, j \in \mathcal{N},
\end{align}

where

\[ f_j(z_j, \theta_j) = \sum_{h=0}^{H} \nabla f_j(x_j(k+h)), + V_j(z_j, \theta_j^*) \\
+ \mathcal{I}(z_j(k+h)|\mathcal{X}_j) + \mathcal{I}(\theta_j^*(k+h)|\Theta_j) \]

The definition of the operational costs and constraints of the interconnected MG makes the above functions $f_j$ for all $j \in \mathcal{N}$, convex, proper and closed. In this context, proper means $f_j$ is finite at least one point and closed means $f_j$ is lower semi-continuous.

**C. Distributed Augmented Lagrangian algorithm**

The Lagrangian function is obtained by relaxing the consensus constraint (11c) in Problem 4. Let us define the Lagrangian multipliers as $\lambda = [\lambda_1 \cdots \lambda_n]$, $\lambda_j = [\lambda_{j1} \cdots \lambda_{jn}], j \in \mathcal{N}$ and $\lambda_{jm} = [\lambda_{jm}(k) \cdots \lambda_{jm}(k+H[k])]$, $\forall m \in N_j$. Here $\lambda_{jm}(k+h) \in \mathbb{R}$ is associated with the constraint $\theta_{jm}(k+h) - \theta_{mm}(k+h)$. We also define

\[ \lambda_{jj}(k+h) = \sum_{m \in N_j} \lambda_{jm}(k+h), \quad \lambda_j = [\lambda_{jj}(k) \cdots \lambda_{jj}(k+H[k])]. \]

The Augmented Lagrangian (AL) for the optimisation problem can then be defined as

\[ L_p(Z, \theta, \lambda) = \sum_{j=1}^{n} f_j(z_j, \theta_j) + \langle \lambda_j^*, \theta_{jc} \rangle - \langle \lambda_{jj}, \theta_{jj} \rangle \]

\[ - \frac{\rho}{2} \| \theta_{jc} - \theta_{jc} \|^2 \]

where $\rho > 0$ is a penalty parameter. The corresponding dual problem associated with the AL is always differentiable which is not guaranteed with the normal Lagrangian. Therefore, the algorithm that is based on AL has faster convergence compared to DD algorithm that is based on the Lagrangian. However, the decomposable structure is no longer preserved in the AL because of the quadratic term. The ADMM [18] reformulates the problem using additional decision variables and solves the AL in two blocks resulting in a Gauss-Seidel pass at each iterate. Thus, applying the ADMM in the current context would result either in a central coordinator to manage the auxiliary variable as in [19] or in multiple communications at each iteration as in [20].

An alternative approach is to replace the original AL with a separable approximate. In this paper, we adopt the distributed AL proposed in [21] which replaces the quadratic penalty by $n$ separable quadratic penalties. The resulting distributed AL algorithm has proven asymptotic convergence and complexity certificates regarding the number of iterates [22].

At MG$_j$, we define a fixed variable $\hat{\theta}_j$ as

\[ \hat{\theta}_j = [\hat{\theta}_{j,j} \text{ vec}(\hat{\theta}_{mj})]_{m \in N_j}, \]

where $\hat{\theta}_{j,j} \in \mathbb{R}^{H_{mj}}$ and $\hat{\theta}_{mj} \in \mathbb{R}^{H}$. Now we define the local augmented Lagrangian function as

\[ L_{\rho}(Z, \theta, \lambda) = \sum_{j=1}^{n} \hat{L}_{\rho}(z_j, \theta_j, \hat{\lambda}_j) \]

\[ \hat{L}_{\rho}(z_j, \theta_j, \hat{\lambda}_j) = f_j(z_j, \theta_j) + \langle \lambda_j, \hat{\theta}_{jc} \rangle - \langle \lambda_{jj}, \theta_{jj} \rangle \]

\[ + \frac{\rho}{2} \| \theta_{jc} - \hat{\theta}_{jc} \|^2 + \sum_{m \in N_j} \frac{\rho}{2} \| \theta_{jj} - \hat{\theta}_{mj} \|^2, \]

where $\hat{\lambda}_j = [\lambda_j \lambda_{jj}]$. The separable approximate for the AL (12) is given as

\[ \hat{L}_{\rho}(Z, \theta, \lambda) = \sum_{j=1}^{n} \hat{L}_{\rho}(z_j, \theta_j, \hat{\lambda}_j). \]

At iterate $\nu$, $\hat{\theta}_j^\nu$ at each MG is updated. We have $\hat{\theta}_j^\nu = \hat{\theta}_j^{\nu-1}$, the phase angles at the neighbours and $\hat{\theta}_{mj}^\nu = \hat{\theta}_{mj}^{\nu-1}$, the duplicated phase angle of MG$_j$. Then the phase angle and the power is updated by solving the the local AL in (13).

\[ z_j^\nu, \hat{\theta}_j^\nu \in \arg \min_{z_j, \theta_j} L_{\rho}(z_j, \theta_j, \hat{\lambda}_j^\nu), \]

\[ \theta_{ja}^{\nu+1} = \hat{\theta}_j^\nu + \tau (\hat{\theta}_j^\nu - \hat{\theta}_j^\nu), \]

where $\tau \in (0, 1/2)$. Now MG$_j$ communicates $\theta_{ja}^{\nu+1}$ and $\theta_{ja}^{\nu+1}$ to its neighbouring MGs and update the dual variables $\lambda_{jj}, \lambda_j$. This is summarised in Algorithm [1].

Algorithm [1] terminates if a predefined number of iterations $\nu_{max}$ or the residual $\epsilon_j^\nu$ at each node $j \in \mathcal{N}$ is reached. At a node $j$, the residual is defined as

\[ \epsilon_j^\nu = \max(||\hat{\theta}_j^\nu - \hat{\theta}_j^\nu||, ||\hat{\theta}_j^\nu - \hat{\theta}_j^\nu||), \]

where $||\hat{\theta}_j^\nu - \hat{\theta}_j^\nu||$ and $||\hat{\theta}_j^\nu - \hat{\theta}_j^\nu||$ is the primal and dual residual.
Algorithm 1 Accelerated distributed AL

Initialisation at time $k \in \mathbb{N}$:
- set $\nu = 0$, $\tau \in (0, 1/2)$, $\rho > 0$
- set $\lambda^0_j, \lambda^{0}_{j,j}, \tilde{\theta}^0_j, \hat{\theta}_j^0$ to zero at every MG, $j \in N$.

Repeat
1) At each MG, fix $\lambda_j^\nu, \lambda^{j,j}, \tilde{\theta}_j^\nu$ and solve (in parallel)
   $$
   z_j^\nu, \tilde{\theta}_j^\nu \in \text{argmin} \ L_j(z_j, \theta_j, \tilde{\theta}_j^\nu, \lambda_j^\nu).
   $$
2) Update local phase angles at each MG
   $$
   \theta_j^{\nu+1} = \theta_j^\nu + \tau (\tilde{\theta}_j^\nu - \theta_j^\nu).
   $$
3) At each MG, $j \in N$: communicate $\theta^{\nu+1}_j$ and $\theta^{j+1}_j$ to all neighbouring MGs.
4) At each MG, $j \in N$: update $\tilde{\theta}_j^{\nu+1}$ using the phase angles received from the neighbours. Update $\lambda^{\nu+1}_j$ and $\lambda^{j,j+1}_j$ according to
   $$
   \lambda^{\nu+1}_j = \lambda^\nu_j + \rho \tau (\theta^{\nu+1}_j - \theta^{j+1}_j),
   $$
   $$
   \lambda^{j,j+1}_j = \lambda^\nu_j + \rho \tau \sum_{m \in N_j} (\theta^{\nu+1}_j - \theta^{j+1}_j).
   $$
5) Update $\nu = \nu + 1$.

Until Terminate if $\nu = \nu_{\text{max}}$ or $\epsilon_j^\nu \leq \epsilon_{\text{term}}$, $\forall j \in N$.

V. CASE STUDY

The proposed distributed MPC based EMS is evaluated in two categories: convergence of the distributed AL algorithm and closed-loop simulation to illustrate the benefits of power transfer between the MGs. We have considered the interconnected MG topology presented in [12] for the case study. As shown in Figure 1, the system consists of four MGs connected by an electrical network. Each MG has a renewable source, a storage device, a conventional generator and local demand. The parameters of the electrical grid, the operational constraints of the units and the weights of the operational cost of this system can be found in [12]. The wind and irradiation data for the RES were provided by [23] and load demand which emulates a realistic load pattern was used.

The EMS provides power values to the units at a sampling time of $T_s = 30 \text{ min}$. The MPC problem in the EMS is formulated with a prediction horizon of 6h, i.e., $H = 12$. A naive persistence forecaster (see, e.g., [24]) is used to predict the the load and renewable infed. The simulations are performed in MATLAB® 2016a. The optimisation problem is formulated with YALMIP [25] and solved with Gurobi 6.5. The analysis was carried out for a period of one week, i.e., 336 sampling points.

A. Computing time

The first stage of the control scheme is solving the islanded MPC problem [1] for four MGs. Using the binary variables found in the first stage we solve the second stage MPC in Problem 3. This is done using the distributed AL Algorithm 1. We have selected $\tau = 0.3$ and $\rho = 10^4$. The parameter $\rho$ can be used to tune the algorithm. A large $\rho$ increases the quadratic penalty in the objective function and a small $\rho$ leads to smaller updates of the dual variables. The current selection of $\rho$ is based on a sensitivity analysis. We compared the convergence of the AL against the dual decomposition [13] algorithm that has similar information exchange at each iterate. We have selected a termination condition of $\epsilon_{\text{term}} = 10^{-5}$.

Table I summarises the maximum and the average number of iterations as well as the computing time per iterate for the AL and the DD algorithm. Both have a similar computing time per iterate. However the distributed AL algorithm has a maximum of 394 iterations which is less than half the number of maximum iteration of DD. As both require one communication with all the neighbours per iterate, the maximum number of communications in the DD algorithm are 869 and in the distributed AL 394. Because of communication delays in physical networks, algorithms with fewer communications are preferred. The fast convergence of the residual is also illustrated in Figure 2. Note in DD the residual [16] is only the dual residual.

<table>
<thead>
<tr>
<th></th>
<th>Computing time per iterate</th>
<th>Number of iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distributed AL</td>
<td>&lt;0.09</td>
<td>124.6</td>
</tr>
<tr>
<td>Dual decomposition</td>
<td>&lt;0.08</td>
<td>407.2</td>
</tr>
</tbody>
</table>

Table I

![Fig. 2. Convergence of distributed AL and dual decomposition at one sample point.](image)

B. Closed-loop simulations

Here we study the closed-loop performance of the interconnected system with three kinds of controllers: 1) centralised two-stage MPC, 2) convex distributed two-stage MPC and 3) islanded MPC. With each of the controllers, the interconnected system was simulated for a period of one week.

The main objective when connecting the MGs is to maximise the infeed of RES. To evaluate this, we use the performance indicator KPI, i.e., the percentage of available renewable power that was used. This indicator is defined as

$$
KPI_{j,r} = \frac{\sum_{k=1}^{M} p_{j,i}(k)}{\sum_{k=1}^{M} w_{j,i}(k)} \cdot 100\%.
$$
for \( j \in N \) and a simulation horizon of \( M = 336 \). \( KPI_{j,r} \)
for each MG and the overall system, i.e., all MGs, is given in Table II. We can observe that KPI increases when trading power compared to the islanded operation. This indicates an increase of renewable infed with the interconnection of MGs.

Next the closed-loop cost incurred using the three controllers as summarised in Table II are discussed. The costs were computed using the same weights as in the objective function. We can observe that the operational cost of the MGs and the overall system decrease when power exchange is allowed. The centralised controller and the convex distributed controller yield a decrease of the costs of 51.1% and 44.5% respectively in comparison to the islanded controller. It can be further observed, that the distributed MPC results in sub-optimal performance compared to the centralised controller. This can also be observed in the renewable infed: KPI, with central controller is 3.2% higher than with the distributed controller. In terms of cost, the distributed solution leads to an increase in costs of 15.6% compared to the islanded solution. This seems reasonable considering that the distributed controller does not have problems with feasibility, scalability and preserves the autonomy of the MGs.

### VI. Conclusions

In this paper we proposed a two-stage distributed MPC based control strategy to operate interconnected MGs. This control strategy preserves the interests and autonomy of the MGs such that an MG in the network trades power only when it is beneficial to it. We proposed a sub-optimal distributed solution for the global mixed-integer problem that always guarantees feasibility. This control strategy does not require a central coordinator to manage the power transfers between MGs. It is realised through reformulation of the distributed MPC as a consensus optimisation on the phase angles of the PCC buses. The optimisation problem is solved with the distributed augmented Lagrangian algorithm that has parallel update scheme and superior convergence properties than dual decomposition. It is shown that distributed MPC based EMS has the potential to increase the overall renewable infed.

Future work will focus on including the forecast errors in distributed robust MPC formulations. Additionally, alternative ways to address integer variables in distributed algorithms shall be investigated.

### References


### Table II

<table>
<thead>
<tr>
<th>KPI</th>
<th>Closed-loop costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central</td>
<td>Distributed</td>
</tr>
<tr>
<td>Central</td>
<td>Distributed</td>
</tr>
</tbody>
</table>

| MG1 | 83.6 | 77.5 | 50.4 | 1.2 | 34.1 | 242.1 |
| MG2 | 74.2 | 71.9 | 66.9 | 101.2 | 110.1 | 128.6 |
| MG3 | 86.4 | 86.5 | 85.7 | 70.6 | 67.0 | 73.1 |
| MG4 | 86.9 | 87.2 | 86.1 | 91.7 | 89.6 | 98.2 |
| Overall | 82.8 | 79.6 | 65.9 | 264.6 | 300.8 | 542.1 |