ABSTRACT: This paper presents a new approach to build extremely light bridges. By using high-strength carbon fibre reinforced plastic, a very light and flexible stress ribbon footbridge having a span of 13 m and a structural height of only 1 mm was built in the lab of the Chair of Conceptual and Structural Design at TU Berlin. To reduce their exceptional high vibration sensitivity, an active vibration control concept was developed and applied to the prototype. This concept includes the embedding of smart actuators. Here, biologically inspired, extremely light and powerful pneumatic muscle actuators are used. To control the first three vertical modes of the bridge by three actuators placed at midspan and at the quarter-points on each side of the handrail a multivariable control concept is developed. This concept is based on a cascaded control structure consisting of subsidiary loops to control the force of the pneumatic muscle actuators and an outer loop to finally control the first three vertical modes of the bridge. The reduced discretized analytical model for the stress ribbon bridge used for model-based control designs is verified by experiments. The comparison between simulation and experiment of the entire closed-loop control after actuator-induced vibrations shows the quality of the models and the control design. The efficiency of the multimodal active vibration control is shown by an enormous increase of damping up to 34 times. Finally, pedestrian-induced vibrations can efficiently be reduced to a comfortable level.

KEY WORDS: multimodal active vibration control; lightweight structure; stress ribbon bridge; footbridge dynamics; pedestrian-induced vibration; pneumatic muscle actuator; velocity feedback control; root locus; time delay

1 INTRODUCTION
Active lightweight design is the answer to today’s material and manufacturing technologies, which allow not only the building of elegant ultra-light and slender bridges; they also lead to sustainable bridges because material, i.e. the use of resources, is minimised. However, lightweight bridges and even more so ultra-light footbridges are very sensitive to pedestrian-induced vibrations as their stiffness and structural damping are very low [1], [11]. Usually, to keep vibrations within acceptable limits additional passive dampers are installed [9]. An alternative approach to ensure structural serviceability is to use active vibration control. In particular this is necessary for ultra-light structures, where the system properties like mass and stiffness become time-variant by changing pedestrian traffic. Here, natural frequencies start to depend on live loads and some passive damping techniques would no longer operate optimally. Furthermore, the natural frequencies of light footbridges are located in a lower frequency range, which allows excitations on several natural frequencies by pedestrians. As a result, vibrations in several modes have to be mitigated. These complex interactions between variable loads and structural response can be effectively controlled only with active systems.

2 CONCEPT OF ACTIVE VIBRATION CONTROL
The applied concept of active vibration control for the stress ribbon bridge is to control vertical modes of the bridge that coincide with the dominant frequencies of pedestrian-induced loads range from 1.34 Hz to 3.75 Hz due to walking, running and jumping. In this paper the first three vertical modes in this range are controlled. Therefore, pneumatic muscle actuators [10] are embedded at midspan and at the quarter-points on each side of the handrail (Figure 1, Figure 2).

Figure 1. Active controlled stress ribbon bridge (Set-up B).
Figure 2. Equipment to control the force of the PMA.
These extremely light pulling-only actuators generate damping forces during the upward movement of the vibrating bridge by inflating the membrane tube of the muscle with compressed air. To compensate the nonlinearities of the actuator system a model-based force control was designed and verified by experiments [1]. The multivariable control concept is based on a cascaded control structure consisting of subsidiary loops to control the force and an outer loop to control the first three modes of the bridge by generating reference forces for the inner loops. The controlled actuator dynamics caused by pressure build-up can be handled as a linear dynamic system (Section 6) and must be considered when designing the outer loop. This control design is done with the root locus method (Subsection 7.1) based on the analytical model of the bridge’s natural mode (Section 3) and the linear actuator dynamics. For each mode to be controlled a velocity feedback control is designed, where the modal velocities are estimated by a Kalman filter using only two measurements (Section 4 and 5). Finally, the modal reference forces are transformed to obtain physical actuator reference forces (Subsection 7.2). Results are shown in Section 8.

3 ANALYTICAL MODEL OF THE BRIDGE FOR MULTIMODAL AND MULTIVARIABLE CONTROL

An analytical plane rigid body model for designing the active vibration control system is developed from the distributed system of the stress ribbon bridge. In order to get a good agreement with experimental data for the modes to be controlled the deck of the bridge is discretized by eight plates, which are pin-jointed coupled and form a sprocket chain. The bearing conditions at the two ends of the chain are fixed in vertical and elastic in horizontal direction. The elastic flexibility modelled by horizontal springs represents the extensional stiffness of the CFRP ribbons, which are not modelled themselves. The control forces acting on the system are generated by three actuators placed at midspan and at the quarter points at handrail level (Section 4). From these points the actuator forces are transferred to the next actuator by pin-jointed bars. Force transfer across the actuators does not occur because the force control for each actuator ensures that only the reference force will be generated (Section 6). The stiffening effect of the railing is omitted as its influence on the modes to be controlled and the controlled system is negligible (Table 2). Also the horizontal vibration of the plate masses is omitted because of small influence on the vertical vibrations. Thus a plane model with seven degrees of freedom respectively generalized coordinates \( q_i \) is developed using Euler-Lagrange equations (cf. Figure 3 and [1]).

For generalized coordinates, the vertical pin-joint coordinates are chosen, except at the bearings. The parameters of the 8-plate model and the real bridge are listed in Table 1. A detailed description of the 8-plate model is given in [1], [8].

Table 1. Parameters of the 8-plate model and the real bridge.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>8-plate</th>
<th>Real bridge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total bridge length [m]</td>
<td>( L_T )</td>
<td>13.05</td>
</tr>
<tr>
<td>Plate length [m]</td>
<td>( L )</td>
<td>1.63</td>
</tr>
<tr>
<td>Plate thickness [m]</td>
<td>( d )</td>
<td>0.10</td>
</tr>
<tr>
<td>Total mass [kg]</td>
<td>( M_i )</td>
<td>4336</td>
</tr>
<tr>
<td>Plate mass [kg]</td>
<td>( m_i )</td>
<td>542</td>
</tr>
<tr>
<td>Mass moment of inertia ([\text{kgm}^2])</td>
<td>( I_{x_i} )</td>
<td>120</td>
</tr>
<tr>
<td>Spring stiffness ([\text{N/m}])</td>
<td>( k )</td>
<td>7 356 000</td>
</tr>
<tr>
<td>Extensional stiffness ([\text{MN}])</td>
<td>( EA )</td>
<td>48</td>
</tr>
<tr>
<td>Pre-stressing force [N]</td>
<td>( P )</td>
<td>300 000</td>
</tr>
<tr>
<td>Handrail post length [m]</td>
<td>( h )</td>
<td>1.09</td>
</tr>
</tbody>
</table>

The resulting seven nonlinear ordinary differential equations of 2\(^{nd}\) order can be linearized at the equilibrium point in case of small amplitudes guaranteed by active vibration control. Therefore, the nonlinear differential equations are derived into a nonlinear state space description defined as follows:

\[
\dot{x} = f(x, u)
\]

(1)

Where \( u = [u_1, u_2, u_3] \) is the input vector and \( u_1 = F_{M1} + F_{M2}, u_2 = F_{M3} + F_{M4}, u_3 = F_{M5} + F_{M6} \) are the input variables each built by actuator forces \( F_{M} \) of two actuators, one on each side. The active control forces in Figure 3 are given by \( AC = u_i/2 \) assuming equal force distribution to the handrail posts. For state vector \( x \in \mathbb{R}^{14×1} \), seven generalized coordinates and their first derivatives are selected:

\[
x = (x_1, x_2, \ldots, x_{13})^T = (z_1, z_1, \ldots, z_6, z_6)^T \quad (2)
\]

The linear state space description in nodal from (3) is obtained by determining the Jacobian matrices (4) at the equilibrium point. The equilibrium point can be calculated by solving the equation \( \dot{x} = f(x, u) = 0 \). Finally, the system matrix \( A \in \mathbb{R}^{14×14} \), the input matrix \( B \in \mathbb{R}^{14×3} \), the output vector is \( y \in \mathbb{R}^{14×1} \) and the output matrix \( C \in \mathbb{R}^{14×14} \) is the identity matrix.

\[
x = A \cdot x + B \cdot u, \quad y = C \cdot x
\]

(3)

Figure 3. 8-plate model and definition of coordinates for plate \( i \).
To verify the developed model, the results of simulation under free vibration are compared with experimental data. In Table 2 the calculated natural frequencies of the 8-plate model as well as the experimental identified natural frequencies of the bridge without railing and the bridge under set-up B (Figure 1) are listed. The first vertical mode shape is symmetric, the second mode shape is asymmetric, etc. Particularly with regard to the modes to be controlled, the variation of the calculated frequencies does not exceed 3.5% and show, that the linearized model describes the natural modes sufficiently exact.

Table 2. Calculated and experimentally (Exp) identified natural frequencies $f_i$ [Hz].

<table>
<thead>
<tr>
<th>No.</th>
<th>8-plate model</th>
<th>Exp w/o railing</th>
<th>Exp set-up B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.33</td>
<td>1.34 (+0.8 %)</td>
<td>1.32 (-0.8 %)</td>
</tr>
<tr>
<td>2</td>
<td>2.54</td>
<td>2.48 (-2.4 %)</td>
<td>2.45 (-3.5 %)</td>
</tr>
<tr>
<td>3</td>
<td>3.85</td>
<td>3.75 (-2.6 %)</td>
<td>3.76 (-2.3 %)</td>
</tr>
<tr>
<td>4</td>
<td>5.34</td>
<td>4.98</td>
<td>4.89</td>
</tr>
<tr>
<td>5</td>
<td>6.98</td>
<td>6.32</td>
<td>6.64</td>
</tr>
<tr>
<td>6</td>
<td>8.67</td>
<td>7.53</td>
<td>8.65</td>
</tr>
<tr>
<td>7</td>
<td>10.09</td>
<td>8.74</td>
<td>9.78</td>
</tr>
</tbody>
</table>

To design a model based controller for specific modes it is more convenient to use the modal state space representation [2]. This representation is obtained from the already derived nodal state space transformation by transformation using the modal matrix $R \in \mathbb{R}^{14 \times 14}$. In the modal description all modes are decoupled:

$$\dot{x}_m = A_m \cdot x_m + B_m \cdot u , \quad y = C_m \cdot x_m , \quad x_m = R^{-1} \cdot x$$

(5)

$$A_m = R^\dagger A R = \text{diag}(A_{m,7} \cdots A_{m,14}), \quad A_{m,i} = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 & -2\zeta_i\omega_i \end{bmatrix}$$

(6)

$$B_m = R^\dagger B = \begin{bmatrix} B_{m,7,1} & B_{m,7,2} & B_{m,7,3} \\ \vdots & \vdots & \vdots \\ B_{m,14,1} & B_{m,14,2} & B_{m,14,3} \end{bmatrix}$$

(7)

$$C_m = CR = [C_{m,7} \cdots C_{m,14}]$$

(8)

where $\omega_i$ is the natural angular frequency and $\zeta_i$ is the modal damping of mode $i$.

4 OPTIMAL PLACEMENT OF SENSORS AND ACTUATORS

There are several options to find proper places for sensors and actuators. For simple set-ups the placement can be found in an ad hoc manner. Aiming at a reduction of the number of sensors and actuators to observe and control several modes a heuristic way will be difficult. To find the optimal locations for a small subset of actuators and sensors from a given large number of actuator and sensor locations the controllability and observability Gramians can be used [2]. Therefore, an analytical model of the system is necessary (cf. Section 3). First, the observability matrices $W_{o,k}$ for $K$ sets of sensors are determined by solving the following equation:

$$A_n^T W_{o,k} + W_{o,k} A_n + (C_i C_n)^T (C_i C_n) = 0,$$

(9)

where $C_i \in \mathbb{R}^{2 \times 14}$ is the sensor configuration matrix of set $k$ defining $z$ nodal states that can be measured. Thus $C_i$ contains only entries that are zero or one. By picking out the “velocity” entry $W_{o,k,ij}$ of mode $i$ of the diagonally dominant observability matrices the standardized observability indices $\tilde{W}_{o,k,ij}$ can be calculated for $K$ sensor configurations under investigation:

$$\tilde{W}_{o,k,ij} = W_{o,k,ij} \cdot 1 / \sqrt{\sum_{i=1}^{K} W_{o,k,ij}^2}$$

(10)

Then the $H_2$-Norm $\|G\|_2$ is used which is the RMS sum of the indices $\tilde{W}_{o,k,ij}$ of the regarded modes $i$:

$$\|G_{o,k}\|_2 \approx \sqrt{\sum_{i=1}^{K} \tilde{W}_{o,k,ij}^2}$$

(11)

To find the optimal set of sensors two aspects are considered: The first three vertical modes have to be observed optimal, that means a high value of $\|G_{o,k}\|_2$. The last four modes have to be observed second best to observe spillover effects. Following these guidelines, thus, the optimal measurement is:

$$y_s = [x_1 \quad x_2 \quad x_7 \quad x_8]^T$$

(12)

where $x_2$ and $x_7$ are obtained by numerical differentiation.

The optimal set of actuators is found by the method called “Simultaneous Placement of Actuators and Sensors” according to Gawronski [2]. Here, only the result is given which is already shown in Figure 3 with actuators placed at midspan and at the quarter points [1]. Once the optimal location for a set of actuators and sensors is found, the controllability and observability of the regarded modes of the set have to be checked because this is not automatically fulfilled by the determination of the Gramians and the used method.

5 KALMAN FILTER TO ESTIMATE MODAL STATES

Based on the modal state space description a Kalman filter [3] is employed to estimate the modal states because they are not directly measurable. By the measurement $y_s$ given in equation (12) the symmetric and asymmetric modes can be observed. The dynamics of the estimator are given by

$$\dot{\hat{x}}_m = A_m \cdot \hat{x}_m + B_m \cdot u + L \left( y_s - C_m \cdot \hat{x}_m \right)$$

(13)

$$\hat{y} = C_m \cdot \hat{x}_m$$

(14)

where $\hat{x}_m \in \mathbb{R}^{14 \times 1}$ are the estimated modal states and $\hat{y}$ is the estimation of the measurement based on $\hat{x}_m$. The matrix $C_m$ selects the estimated states $\hat{x}_1$, $\hat{x}_2$, $\hat{x}_7$ and $\hat{x}_8$ out of the
vector $\mathbf{C}_m \cdot \dot{\mathbf{x}}_m$. The convergence of $\dot{\mathbf{x}}_m \rightarrow \mathbf{x}_m$ is determined by the observer gain $L$. This gain is found by minimizing a quadratic cost function. The estimated nodal states $\hat{\mathbf{y}}$ are given by equation (14).

6 FORCE CONTROL

To handle the nonlinearity of the actuator system a nonlinear force controller is designed based on exact linearization methods. Therefore, a nonlinear analytical model of the pneumatic muscle, the valve characteristic and the pressure build-up is derived. Detailed information about modelling of pneumatic actuators including mass flow, pressure dynamics and force characteristics are given in literature [4], [5].

The linearization method is applied to render the nonlinear actuator dynamics linear and stable by a nonlinear control law. The nonlinear actuator model and the nonlinear control law are given in [1], [12]. The resulting linear actuator dynamics can be described by a transfer function where $\omega$ are given in [1], [12].

The force control was simulated and verified experimentally in [1]. A linear actuator dynamics with $T_f = 0.05 \text{ s}$ and $a_0 = 3.18 \text{ Hz}$ respectively could be obtained as long as the supply pressure is constant also under load changes, no saturation of the input variable occurs and the sampling rate of the digital controller realization is high enough.

7 CONTROL DESIGN FOR MULTIMODAL VIBRATION CONTROL

In this section the control parameters for the superior loop of the cascaded control structure are determined in two steps. First, the control parameters for each modal velocity feedback control are calculated by applying the root locus method. Therefore, the actuator dynamics (15) and a controller time delay are considered in the closed loop. The time delay of the closed loop system for each mode, given by equation (19), are calculated using a numerical algorithm [6].

Each root locus is plotted for a fix dimensionless time delay $\tau = T_{d,i} / T_m$ against variable gains $k_{\text{act},i} = \{0.0, 0.5, \ldots\}$, where $T_m$ is the natural period. By varying the dimensionless time delay $\tau$ a set of root locus plots is obtained illustrated e.g. for the 3rd mode in Figure 4.

7.1 Root locus method considering actuator dynamics

Input for each modal velocity feedback control design is the estimated modal velocity $\hat{\dot{x}}_m$, output is the modal control variable $\hat{u}_m$, $i = 1...3$. The transfer function $G_{\text{SISO},m}(s)$ of mode $i$ between the single modal input and the single modal output (17) is derived using the Laplace transform of the modal system matrix $A_m$ given by equation (6), whereas $B_m$ is standardized to one and from $C_m$ only the velocity entry is used:

$$ G_{\text{SISO},m}(s) = \frac{s}{s^2 + 2\zeta_i \omega_i s + \omega_i^2} $$

By considering the linear actuator dynamics and controller dynamics $\hat{K}_m(s)$ and neglecting the observer dynamics, the characteristic equation of the closed loop is given by

$$ 1 + G_{\text{SISO},m}(s) \cdot \hat{K}_m(s) \cdot \hat{K}_m(s) = 0 $$

where $\hat{K}_m(s) = k_{\text{act},i} \cdot \mathbf{C}_m \cdot \mathbf{x}_m$ includes the gain $k_{\text{act},i}$ for velocity feedback control and the time delay $T_{d,i}$. To find the optimal control parameters of $k_{\text{act},i}$ and $T_{d,i}$ the poles $\hat{s}_{\text{SISO},i}$ of the closed loop system for each mode, given by equation (19), are calculated using a numerical algorithm [6].

$$ \hat{s}_{\text{SISO},i} \left( T_{d,i}, k_{\text{act},i}, \omega_i, \zeta_i \right) = -\frac{\partial \hat{\omega}_i}{\partial \zeta_i} \mp i \frac{\partial \hat{\omega}_i}{\partial \tau} \sqrt{1 - \frac{\hat{\omega}_i^2}{\partial \tau}} $$

Each root locus is plotted for a fix dimensionless time delay $\tau = T_{d,i} / T_m$ against variable gains $k_{\text{act},i} = \{0.0, 0.5, \ldots\}$, where $T_m$ is the natural period. By varying the dimensionless time delay $\tau$ a set of root locus plots is obtained illustrated e.g. for the 3rd mode in Figure 4.

Figure 4. Root locus of the 3rd mode.

It can be observed that with increasing values of time delay the poles cross the unstable right complex plane and move back into the stable left complex plane. Optimal control parameters can be found by following two aims: First, by maximizing the damping of the controlled system $\zeta$, or second, by finding the controller setting that minimizes the control effort for a given value of damping. Here, the optimal parameters are found by aiming for maximum damping. The optimal control parameters are listed in Table 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>1st mode</th>
<th>2nd mode</th>
<th>3rd mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\omega}_{i,\text{opt}}$ [rad/s]</td>
<td>10.14</td>
<td>16.84</td>
<td>22.93</td>
</tr>
<tr>
<td>$\hat{\zeta}_{i,\text{opt}}$ [-]</td>
<td>0.32</td>
<td>0.13</td>
<td>0.15</td>
</tr>
<tr>
<td>$\tau_{i,\text{opt}}$ [-]</td>
<td>0.00</td>
<td>0.90</td>
<td>0.85</td>
</tr>
<tr>
<td>$T_{d,i,\text{opt}}$ [s]</td>
<td>0.00</td>
<td>0.35</td>
<td>0.22</td>
</tr>
<tr>
<td>$k_{\text{act},i,\text{opt}}$ [Ns/m]</td>
<td>6.00</td>
<td>2.50</td>
<td>5.00</td>
</tr>
</tbody>
</table>
The gains $k_{w,v}$ are found assuming a non-saturated control signal (force). Actually, due to the nature of the chosen actuator only limited unidirectional (positive) control forces can be applied. Simulations have shown that the practically applied gain should be doubled to obtain the above given optimal control parameters. In addition, the stability can be studied theoretically by the Popov Criterion [7]. Finally, the vector of modal control variables is given by:

$$
\begin{bmatrix}
    u_{w1,v} \\
    u_{w2,v} \\
    u_{w3,v}
\end{bmatrix} = 
\begin{bmatrix}
    -2 \cdot k_{w1,v,\text{opt}} & 0 \\
    0 & -2 \cdot k_{w2,v,\text{opt}} \\
    0 & -2 \cdot k_{w3,v,\text{opt}}
\end{bmatrix}
\begin{bmatrix}
    \dot{x}_{w1}(t-T_{d,w1,v,\text{opt}}) \\
    \dot{x}_{w2}(t-T_{d,w2,v,\text{opt}}) \\
    \dot{x}_{w3}(t-T_{d,w3,v,\text{opt}})
\end{bmatrix}
$$

(20)

7.2 Transformation of modal controlled variables into actuator forces

The modal control variables are connected through the modal input matrix $B_m$, equation (7), with the nodal input variables:

$$u_m = B_m \cdot u$$

(21)

To get a correlation between the reduced modal control variables $u_{m,\text{red}} = \begin{bmatrix} u_{w1,v} & u_{w2,v} & u_{w3,v} & u_{e1,v} \end{bmatrix}$, equation (20) and the vector of nodal input variables, a transfer matrix $T_{\text{red}}$ has to be found by using the modal input matrix and a selection matrix $C_T$.

$$u_{m,\text{red}} = C_T \cdot B_m \cdot u = B_{m,\text{red}} \cdot u$$

(22)

The selection matrix is given by

$$C_T = \begin{bmatrix}
    : & : & : & : & : & 1 & 0 & 0 & 0 & 0 \\
    : & : & : & : & : & 0 & 0 & 0 & 0 & 1 \\
    : & : & : & : & : & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
$$

(23)

Due to three modes controlled by three actuators the solution to determine the transfer matrix is given by:

$$T_{\text{red}} = (B_{m,\text{red}})^{-1}$$

(24)

Finally, the input vector for multivariable control is obtained:

$$u = T_{\text{red}} \cdot u_{m,\text{red}}$$

(25)

8 NUMERICAL SIMULATION AND EXPERIMENTAL VERIFICATION

8.1 Details for simulation and implementation of the controller

First, simulations were carried out using the open source software package Scilab / Scicos for numerical computation (http://www.scilab.org, http://www.scicos.org). In order to have a good comparison between the results of simulation and experiment defined actuator forces were introduced as excitation signal. The multimodal excitation by defined actuator forces causes velocity amplitudes of $\dot{x}_{w1} \approx 0.1 \text{ m/s}$, $\dot{x}_{w2} \approx 0.2 \text{ m/s}$ and $\dot{x}_{w3} \approx 0.2 \text{ m/s}$. Afterwards the vibrations were controlled and reduced by actuator forces in the range of $100 \text{ N} \leq F_{m} \leq 3000 \text{ N}$ for each actuator. The actuator dynamics was considered as described in Section 6.

For the experiment the control structure was implemented in a real-time environment with a sampling frequency of 100 Hz. The software code was generated and compiled from Scicos block diagrams. Communication to devices was provided by the HART Toolbox (http://hart.sourceforge.net). The control structure for the experiment is shown in Figure 5. The force control represents the nonlinear control law given in [1], where the force $F_{M,\text{act}}$, the pressure $P_{M,\text{act}}$ and the contraction length $s_{\text{d,act}}$ of the pneumatic muscle actuator are measured. Alternatively, the contraction length can be estimated $(s_{\text{d,act}})$ by an observer using the estimated nodal states $\hat{y}$ [1]. The measured state vector for the Kalman filter is $y$. In Figure 5, the supply pressure is $p$, the mass flow is $m$, and $u_e$ is the reference position of the proportional directional control valve [10].

![Control structure for experiment.](image-url)
8.2 Results

First, the estimated states $\hat{y}$ are verified by an experiment (Exp), in Figure 6 and Figure 7. As a result, the states are estimated by the Kalman filter in good agreement to the true measured values. The results of simulation (Sim) and experiment (Exp) after excitation are shown in the following Figures. In Figure 8 the reference force $F_{M1/2,\text{ref}}$ and the estimated force $F_{M1/2,\text{est}}$ of simulation and experiment as well as the measured force $F_{M1,\text{act}}$ are plotted. It can be observed that the measured force follows the expected linear actuator dynamics $F_{M1/2,\text{est}}$ for the experiment in good agreement. Thus, the estimated modal position and velocity signal of the experiment follow the simulation results as shown in Figure 9 to Figure 11 for the first three modes. In higher modes no significant spillover effect could be observed [1].
To show the efficiency of the developed control strategy the decaying acceleration amplitudes of the experiments after excitation for a single mode without (w/o) control and with control are compared. In Figure 12, the acceleration signals of the first mode are plotted for instance. An upper and lower exponential regression curve is plotted to quantify the damping effect by determining the exponent of the upper curve $b_u$ and lower curve $b_l$. Hence, logarithmic decrements without control $\Lambda_i$ and with control $\Lambda_{i,AVC}$ are calculated by the following equations:

$$\Lambda_i \approx \left( \frac{b_u + b_l}{2} \right) / f_i \quad (26)$$

$$\Lambda_{i,AVC} \approx \left( \frac{2\pi \cdot (b_u + b_l)}{2} \right) / \tilde{\omega}_{opt} \quad (27)$$

The identified values for each mode as well as the more common modal damping $\zeta_i = \Lambda_i / 2\pi$ are listed in Table 4. The controlled response of the first mode decays about 34 times faster, the second mode about 16 times and the third mode about 5 times. The decreasing damping efficiency with increasing natural frequencies can be explained by the limited dynamics of the force control (Section 6).

Table 4. Logarithmic decrement and modal damping without control and with active vibration control (AVC) and increase factor of damping.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Log. decrement [-]</th>
<th>Modal damping [%]</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>0.0394</td>
<td>0.63</td>
<td>17.44</td>
</tr>
<tr>
<td>2nd</td>
<td>0.0490</td>
<td>0.78</td>
<td>11.40</td>
</tr>
<tr>
<td>3rd</td>
<td>0.0798</td>
<td>1.27</td>
<td>6.54</td>
</tr>
</tbody>
</table>

9 CONCLUSION

Multimodal and multivariable active vibration control for a stress ribbon bridge with extremely light pneumatic muscle actuators was investigated. Therefore, an analytical bridge model with a description of the actuator forces acting on the bridge was developed. The need for this model was demonstrated several times: First, the optimal placement of sensors and actuators was found. Second, by designing a Kalman filter to estimate modal states, the number of sensors could be reduced. Third, the velocity feedback control could be designed model-based by the modal state space description of the bridge model. The proposed cascaded control structure offers the possibility to subdivide complex control algorithm, here, the nonlinear behaviour of the actuator system and the active vibration control. This control structure is efficient as long as the dynamics of the inner loop is faster than the outer loop. The modal velocity feedback control design allows an independent control for each mode. Thus, the control performance of the 2nd and 3rd mode could be individually improved by choosing a time delayed controller.

Finally, the active system was implemented in a real-time environment on the stress ribbon bridge. The results of experiment showed the quality of the models and control rules. The comparison of the decaying acceleration amplitudes without and with control showed the efficiency of the developed control system. In future, the performance of the controller reducing pedestrian-induced vibrations will be investigated.

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