Hierarchical distributed model predictive control of interconnected microgrids

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Goals of the energy transition

Today

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Future

Motivation for microgrids (MGs)

Increasing infeed from renewable energy sources
→ change in power system structure
→ increasing fluctuations

Microgrids

- Idea: partition grid into microgrids
- Part of a larger network
- Local dispatch of generation, consumption and storage
- Operated grid-connected or in islanded mode
Interconnected MGs

Questions

- How to operate interconnected MGs in a distributed way?
- What are pros and cons of a distributed operation compared to
  - a central controller and to
  - a system of islanded MGs?
Single microgrid model: variables

Variables

Power \( p_i = [p_{t,i}, p_{s,i}, p_{r,i}, p_{g,i}]^T \)

Control input \( u_i = [\delta_{t,i}, u_{t,i}, u_{s,i}, u_{r,i}]^T \)

Stored energy \( x_i \)

Uncertain input \( w_i = [w_{d,i}, w_{r,i}]^T \)
Single microgrid model: constraints

Stored energy of MG $i$

$$x_i(k+1) = x_i(k) - T_s u_{s,i}(k)$$

$$= f_i(x_i(k), u_i(k))$$

$$x_i(k+1) \in [\underline{x}_i, \overline{x}_i] = \mathcal{X}_i$$

Power of MG $i$

$$[p_{t,i}(k), p_{s,i}(k), p_{r,i}(k), p_{g,i}(k)]^T = [u_{t,i}(k), u_{s,i}(k), u_{r,i}(k),$$

$$- (u_{t,i}(k) + u_{s,i}(k) + u_{r,i}(k) + w_{d,i}(k))]^T$$

$$= g_i(u_i(k), w_i(k))$$

$$[(\delta_{t,i}(k), u_{t,i}(k)), u_{s,i}(k), u_{r,i}(k)]^T \in ((0, 0) \cup (1, [\underline{p}_{t,i}, \overline{p}_{t,i}]]) \times [\underline{p}_{s,i}, \overline{p}_{s,i}]$$

$$\times [\underline{p}_{r,i}, \min \{\overline{p}_{r,i}, w_{r,i}(k)\}] = \mathcal{U}_i(k)$$
Single microgrid model: operational costs

Costs of MG $i$

Overall
$$\ell_i(z_i) = \ell_{t,i}(z_i) + \ell_{r,i}(z_i) + \ell_{s,i}(z_i) + \ell_{g,i}(z_i),$$
with $z_i = [\delta_{t,i}, u_{t,i}, u_{s,i}, u_{r,i}, p_{g,i}]^T$

Thermal
$$\ell_{t,i}(z_i) = c_{t,i}\delta_{t,i} + c'_{t,i}u_{t,i} + c''_{t,i}(u_{t,i})^2$$

RES
$$\ell_{r,i}(z_i) = c_{r,i}(\bar{p}_{r,i} - u_{r,i})^2$$

Storage
$$\ell_{s,i}(z_i) = c_{s,i}(u_{s,i})^2$$

Grid coupl.
$$\ell_{g,i}(z_i) = c'_{g,i}p_{g,i} + c''_{g,i}|p_{g,i}|$$
Grid model: variables & constraints

\[
p_E = \begin{bmatrix} p_{E,1} \\ p_{E,2} \\ p_{E,3} \\ p_{E,4} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2/3 & 1/3 \\ 0 & 0 & 1/3 & 2/3 \\ 0 & 0 & -1/3 & 1/3 \end{bmatrix} \begin{bmatrix} p_{g,1} \\ p_{g,2} \\ p_{g,3} \\ p_{g,4} \end{bmatrix}
\]

Power of grid

\[p_g(k) \in \mathbb{P}_E = \{p_g \in [p_{\underline{g}}, p_{\overline{g}}] | p_E \leq Fp_g \leq p_E \wedge 0 = 1^T_4 p_g\}\]
Grid model: operational costs

\[
\begin{bmatrix}
p_{\varepsilon,1} \\
p_{\varepsilon,2} \\
p_{\varepsilon,3} \\
p_{\varepsilon,4}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 2/3 & 1/3 \\
0 & 0 & 1/3 & 2/3 \\
0 & 0 & -1/3 & 1/3
\end{bmatrix}
\begin{bmatrix}
p_{g,1} \\
p_{g,2} \\
p_{g,3} \\
p_{g,4}
\end{bmatrix}
\]

Costs of transmission network

\[
l_{\varepsilon}(p_{g}) = (Fp_{g})^T \text{diag}(c_{\varepsilon,1}, \ldots, c_{\varepsilon,4})(Fp_{g})
\]
Model predictive control (MPC)

**Problem 1. Mixed integer quadratic MPC**

\[
\min_{u_1(j), \ldots, u_4(j)} \sum_{j=k}^{k+K-1} \gamma(j-k) \left( l_E(p_g(j)) + \sum_{i=1}^{4} l_i(z_i(j)) \right)
\]

\[\text{s.t.} \]
\[
p_i(j) = g_i(u_i(j), w_i(j)),
\]
\[
u_i(j) \in U_i(j),
\]
\[
x_i(j+1) = f_i(x_i(j), u_i(j)), \ \text{with} \ x_i(k) = x_i^k,
\]
\[
x_i(j+1) \in X_i,
\]
\[
p_g(j) \in P_E,
\]
\[
\forall j = k, \ldots, k+K-1, \ \forall i = 1, \ldots, 4
\]
Model predictive control (MPC)

\[ \min_{u_1(k), \ldots, u_4(k)} \sum_{j = k}^{k+K-1} \ell_E(p_g(j)) + \sum_{i = 1}^{4} \ell_i(z_i(j)) \]

s.t. \[
\begin{align*}
    p_i(j) &= g_i(u_i(j), w_i(j)), \\
    x_i(k+1) &= f_i(x_i(k), u_i(k)) \\
    x_i(k) &= x_k_i,
\end{align*}
\] 

\[ \forall j = k, \ldots, k+K-1, \quad \forall i = 1, \ldots, 4 \]

Uncertain \( w \)

Input \( u \)

State \( x \)

Disturbance \( w_k \)

Forecast

Prediction \( w_1(j), \ldots, w_4(j) \) \( j = k, \ldots, k+K-1 \)

Input \( u_1(k), \ldots, u_4(k) \)

Measurement \( x^k \)

Microgrid

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Problem 1. Mixed integer quadratic MPC

\[
\begin{align*}
\min_{u_1(j), \ldots, u_4(j)} & \quad \sum_{j=k}^{k+K-1} \gamma(j-k) (\ell E(p_g(j)) + \sum_{i=1}^{4} \ell_i(z_i(j))) \\
\text{s.t.} & \quad p_i(j) = g_i(u_i(j), w_i(j)), \\
& \quad u_i(j) \in U_i(j), \\
& \quad x_i(j+1) = f_i(x_i(j), u_i(j)), \text{ with } x_i(k) = x_i^k, \\
& \quad x_i(j+1) \in X_i, \\
& \quad p_g(j) \in P_E, \\
& \quad \forall j = k, \ldots, k + K - 1, \forall i = 1, \ldots, 4
\end{align*}
\]

with \( U_i(k) = ((0, 0) \cup (1, [p_{t,i}, \bar{p}_{t,i}])) \times [p_{s,i}, \bar{p}_{s,i}] \times [p_{r,i}, \min\{\bar{p}_{r,i}, w_{r,i}(k)\}] \)
Problem 2. Quadratic convex MPC

\[
\begin{align*}
\min_{u_1(j),\ldots,u_4(j)} & \quad \sum_{j=k}^{k+K-1} \gamma(j-k) (\ell \epsilon(p_g(j)) + \sum_{i=1}^{4} \ell_i(z_i(j))) \\
\text{s.t.} & \\
& p_i(j) = g_i(u_i(j), w_i(j)), \\
& u_i(j) \in U_i^c(j), \\
& x_i(j+1) = f_i(x_i(j), u_i(j)), \text{ with } x_i(k) = x_i^k, \\
& x_i(j+1) \in X_i, \\
& p_g(j) \in P_{\epsilon}, \\
& \forall j = k, \ldots, k+K-1, \forall i = 1, \ldots, 4
\end{align*}
\]

with \[
U_i^c(k) = \{(\delta_{t,i}, u_{t,i}) | \delta_{t,i} \in [0, 1], u_{t,i} \in [\underline{p}_{t,i}, \delta_{t,i}, \overline{p}_{t,i} \delta_{t,i}]\}
\]
\[
\times [\underline{p}_{s,i}, \overline{p}_{s,i}] \times [\underline{p}_{r,i}, \min\{\overline{p}_{r,i}, w_{r,i}(k)\}]
\]
Problem 3. Separable quadratic convex MPC

\[
\min_{u_1(j),\ldots,u_4(j)} \sum_{j=k}^{k+K-1} \gamma(j-k) (\ell \epsilon (\hat{p}_g(j)) + \sum_{i=1}^{4} l_i(z_i(j)))
\]

s.t.

\[p_i(j) = g_i(u_i(j), w_i(j)), \quad (1)\]

\[u_i(j) \in \mathcal{U}_i^c(j), \quad (2)\]

\[x_i(j+1) = f_i(x_i(j), u_i(j)), \text{ with } x_i(k) = x_i^k, \quad (3)\]

\[x_i(j+1) \in \mathcal{X}_i, \quad (4)\]

\[\hat{p}_g(j) \in \mathcal{P}_\epsilon, \quad (5)\]

\[0 = p_g(j) - \hat{p}_g(j), \quad (6)\]

\[\forall j = k, \ldots, k+K-1, \forall i = 1, \ldots, 4\]
Alternating direction method of multipliers (ADMM)

- Problem

\[
\min_{x, \hat{x}} f(x) + g(\hat{x})
\]
\[
s.t. \quad 0 = x - \hat{x}
\]

- Augmented Lagrangian

\[
\mathcal{L}(x, \hat{x}, \lambda) = f(x) + g(\hat{x}) + \lambda^T(x - \hat{x}) + \frac{\rho}{2}\|x - \hat{x}\|_2^2
\]

- ADMM

Minimize \(x\): \(x^{l+1} \in \arg\min_x \mathcal{L}(x, \hat{x}^l, \lambda^l)\)

Minimize \(\hat{x}\): \(\hat{x}^{l+1} \in \arg\min_{\hat{x}} \mathcal{L}(x^{l+1}, \hat{x}, \lambda^l)\)

Update dual: \(\lambda^{l+1} = \lambda^l + \rho(x^{l+1} - \hat{x}^{l+1})\)
Augmented Lagrangian of Problem 3

\[ \mathcal{L}(z(k), \hat{p}_g(k), \Lambda(k)) = \sum_{j=k}^{k+K-1} \left( \gamma^{(j-k)}(l_E(\hat{p}_g(j)) + \sum_{i=1}^{4} l_i(z_i(j))) ight. 
+ \left. \sum_{i=1}^{4} (\lambda_i(j)(p_{g,i}(j) - \hat{p}_{g,i}(j)) + \frac{\rho}{2} \|p_{g,i}(j) - \hat{p}_{g,i}(j)\|_2^2) \right) \]

with

\[ z_i(k) = [z_i(k)^T, \ldots, z_i(k+K-1)^T]^T \text{ for } i = 1, \ldots, 4 \]
\[ z(k) = [z_1(k)^T, \ldots, z_4(k)^T]^T \]
\[ \hat{p}_g(k) = [\hat{p}_g(k)^T, \ldots, \hat{p}_g(k+K-1)^T]^T \]
\[ \lambda(j) = [\lambda_1(j), \ldots, \lambda_4(j)]^T \text{ for } j = k, \ldots, k+K-1 \]
\[ \Lambda(k) = [\lambda(k)^T, \ldots, \lambda(k+K-1)^T]^T \]
ADMM used to solve Problem 3

At every MG $i = 1, \ldots, 4$, iteration $l + 1 = 1, \ldots, l_{\text{max}}$

Solve $z_i^{l+1}(k) \in \arg\min_{z_i(k)} \sum_{j=k}^{k+K-1} (\gamma^{(j-k)} l_i(z_i(j)))$

\begin{equation}
+ \lambda_i^l(j)p_{g,i}(j) + \frac{\rho}{2} \| p_{g,i}(j) - \hat{p}_{g,i}^l(j) \|^2_2 \end{equation}

s.t. (1)–(4) $\forall j = k, \ldots, k + K - 1.$

At central entity, iteration $l + 1 = 1, \ldots, l_{\text{max}}$

Solve $\hat{p}_{g}^{l+1}(k) \in \arg\min_{\hat{p}_{g}(k)} \sum_{j=k}^{k+K-1} (\gamma^{(j-k)} l_{\mathcal{E}}(\hat{p}_{g}(j)))$

\begin{equation}
- \hat{p}_{g}(j)^T \lambda(j)^l + \frac{\rho}{2} \| p_{g}^{l+1}(j) - \hat{p}_{g}(j) \|^2_2 \end{equation}

s.t. $\hat{p}_{g}(j) \in \mathcal{P}_{\mathcal{E}} \forall j = k, \ldots, k + K - 1.$

Update $\lambda_i^{l+1}(j) = \lambda_i^l + \rho (p_{g,i}^{l+1}(j) - \hat{p}_{g,i}^{l+1}(j))$, $\rho \in \mathbb{R}_{>0}$

$\forall j = k, \ldots, k + K - 1, \forall i = 1, \ldots, I.$
Algorithm 1. Distributed MPC

(1) Initialization at time \( k \in \mathbb{N} \):
   - **MGs:** For \( i, \ldots, I \), measure \( x_i(k) = x_i^k \)
   - **Central Entity:** Define grid constraints \( \mathbb{P}_\mathcal{E} \)

(2) Main Loop (ADMM): For \( l = 1, \ldots, (l_{\max} - 1) \in \mathbb{N} \)
   (i) **MGs:** For \( i = 1, \ldots, I \),
       - Solve (7) in parallel
       - Send \( p_{g,i}^{l+1}(k), \ldots, p_{g,i}^{l+1}(k+K-1) \) to the central entity
   (ii) **Central Entity:**
       - Solve (8)
       - Update the Lagrange multipliers (9)
       - Communicate \( \hat{p}_{g,i}^{l+1}(k), \ldots, \hat{p}_{g,i}^{l+1}(k+K-1) \) and \( \lambda_i^{l+1}(k), \ldots, \lambda_i^{l+1}(k+K-1) \) to all MGs \( i = 1, \ldots, I \)

(3) Mixed integer update:
   For \( l = l_{\max} \), for all MGs \( i = 1, \ldots, I \)
   - Solve (7) where \( \mathbb{U}_i^c(j) \) is replaced by \( \mathbb{U}_i(j) \) and the constraints
     \[ p_{g,i}(j) - \hat{p}_{g,i}^{l_{\max}}(j) = 0 \]
     are added for \( j = k, \ldots, k+K-1 \)

(4) Application of \( u_i^{l_{\max}+1}(k) \), increment \( k \) and go to (1)
Algorithm 1. Distributed MPC

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\textbf{Algorithm 1. Distributed MPC}

Motivation Model Controller Case study Conclusions References

\textbf{Motivation}

\textbf{Model}

\textbf{Controller}

\textbf{Case study}

\textbf{Conclusions}

\textbf{References}

\textbf{Algortihm 1. Distributed MPC}

\begin{align*}
z_1^2(k) &= \arg\min_{z_1(k)} \sum_{j=1}^{k+K-1} (\gamma(j-k)\ell_1(z_1(j))) \\
&\quad + \lambda_1^1(j)p_{g,1}(j) + \frac{\rho}{2}\|p_{g,1}(j) - \hat{p}_{g,1}^2(j)\|^2 \\
&\text{s.t. (1)-(4) } \forall j = k, \ldots, k+K-1.
\end{align*}

\begin{align*}
z_2^2(k) &= \arg\min_{z_2(k)} \sum_{j=1}^{k+K-1} (\gamma(j-k)\ell_2(z_2(j))) \\
&\quad + \lambda_1^1(j)p_{g,2}(j) + \frac{\rho}{2}\|p_{g,2}(j) - \hat{p}_{g,2}^2(j)\|^2 \\
&\text{s.t. (1)-(4) } \forall j = k, \ldots, k+K-1.
\end{align*}

\begin{align*}
p_3^2(k) &= \arg\min_{p_3(k)} \sum_{j=1}^{k+K-1} (\gamma(j-k)\ell_3(p_3(j))) \\
&\quad - \hat{p}_{g,1}(j)\lambda(j) + \frac{\rho}{2}\|p_{g,1}(j) - \hat{p}_{g,1}^2(j)\|^2 \\
&\text{s.t. } \hat{p}_{g,1}(j) \in \mathcal{P}_c \forall j = k, \ldots, k+K-1.
\end{align*}

\begin{align*}
\lambda_1^2(k) &= \lambda_1^1(j) + \rho (p_{g,1}(j) - \hat{p}_{g,1}^2(j)) \\
&\forall j = k, \ldots, k+K-1, \forall i = 1, \ldots, 4.
\end{align*}

\begin{align*}
z_4^2(k) &= \arg\min_{z_4(k)} \sum_{j=1}^{k+K-1} (\gamma(j-k)\ell_4(z_4(j))) \\
&\quad + \lambda_1^1(j)p_{g,4}(j) + \frac{\rho}{2}\|p_{g,4}(j) - \hat{p}_{g,4}^2(j)\|^2 \\
&\text{s.t. (1)-(4) } \forall j = k, \ldots, k+K-1.
\end{align*}

\begin{align*}
z_3^2(k) &= \arg\min_{z_3(k)} \sum_{j=1}^{k+K-1} (\gamma(j-k)\ell_3(z_3(j))) \\
&\quad + \lambda_1^1(j)p_{g,3}(j) + \frac{\rho}{2}\|p_{g,3}(j) - \hat{p}_{g,3}^2(j)\|^2 \\
&\text{s.t. (1)-(4) } \forall j = k, \ldots, k+K-1.
\end{align*}
Case study

Motivation Model Controller Case study Conclusions References

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Case study (islanded MGs)
Case study (islanded MGs)

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Case study (islanded MGs)(interconnected MGs)

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Case study (interconnected MGs)
Case study (interconnected MGs)
Case study
## Quantitative comparison

<table>
<thead>
<tr>
<th></th>
<th>Dist. MPC (Alg. 1)</th>
<th>Central mixed integer MPC</th>
<th>Islanded MPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Renewable share</td>
<td>89.1%</td>
<td>89.3%</td>
<td>69.2%</td>
</tr>
<tr>
<td>Avg. closed loop costs</td>
<td>9.84</td>
<td>9.83</td>
<td>10.99</td>
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Conclusions

- **ADMM based strategy**
  - performs essential optimization step autonomously by MGs and
  - allows to hide cost function and structure of local MGs.

- **Interconnection significantly**
  - increases renewable share and
  - decreases overall operation cost.

- **Case study:** closed-loop cost increase less than 1% compared to global MIQP solution.
