

# Magnetometer-free inertial motion tracking of arbitrary joints with range of motion constraints

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**Abstract:** In motion tracking of connected multi-body systems Inertial Measurement Units (IMUs) are used in a wide variety of applications, since they provide a low-cost easy-to-use method for orientation estimation. However, in indoor environments or near ferromagnetic material the magnetic field is inhomogeneous which limits the accuracy of tracking algorithms using magnetometers. Methods that use only accelerometers and gyroscopes on the other hand yield no information on the absolute heading of the tracked object. For objects connected by rotational joints with range of motion constraints we propose a method that provides a magnetometer-free, long-term stable relative orientation estimate based on a non-linear, window-based cost function. The method can be used for real-time estimation as well as post-processing. It is validated experimentally with a mechanical joint and compared to other methods that are used in motion tracking. It is shown that for the used test object, the proposed methods yields the best results with a total angle error of less than  $4^\circ$  for all experiments.

*Keywords:* information and sensor fusion, inertial measurement units, inertial sensors, motion tracking, state estimation, magnetometer-free inertial motion tracking, exploitation of kinematic constraints, moving horizon estimation

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## 1. INTRODUCTION

The tracking of orientations and positions of objects in three-dimensional space is an integral part of various control applications. Many mechanical systems consist of multiple connected objects of which the individual orientations must be known. There exist numerous different approaches for orientation tracking reaching from optical systems, over mechanical solutions to inertial sensors. Inertial Measurement Units (IMUs) have the advantage of being small, low-cost, have a wide field of application and they require no direct interaction with the object of interest. Therefore they are used in a wide variety of robotic and biomedical applications Wong et al. (2015); Miller et al. (2004); Fong and Chan (2010). IMUs usually consist of 3D accelerometers, 3D gyroscopes and 3D magnetometers. Sensor fusion of these measurements yields the orientation of the sensor with respect to a fixed inertial frame. This common inertial frame is necessary to determine the relationships between the individual bodies of a kinematic chain. However, 9D sensor fusion only yields accurate orientation estimates if the magnetic field is homogeneous. In indoor environments and near ferromagnetic material the magnetic field is known to be highly disturbed and inhomogeneous Subbu et al. (2013); de Vries et al. (2009); Shu et al. (2015);

Grand and Thrun (2012). This is crucial for most robotic and biomedical applications and makes conventional 9D sensor fusion inapplicable Salchow-Hömmen et al. (2019). Without magnetometers, the heading component of the orientation is unknown. Without heading information, the estimated orientations of connected bodies cannot be used to determine relative orientations, joint angles or positional relationships. One approach by Salchow-Hömmen et al. (2019) is to omit the magnetometer readings and determine the heading by predefined initial poses. It is shown that the results are better than conventional 9D sensor fusion. However, due to drift this method only produces good results for short-term experiments ( $< 30$  s) and is not long-term stable. Therefore, current research aims to exploit the kinematic relationships between the connected bodies to obtain heading information without using magnetometers or predefined initial poses. For joints with one degree of freedom, there exist methods to calculate the joint angles analytically without the use of magnetometers Cooper et al. (2009); Seel et al. (2014). Recently a method was published which exploits the kinematic relationships with an orientation-based constraint Laidig et al. (2017) to estimate the relative heading of two bodies connected by a one-dimensional joint. For joints with two-degrees of freedom, we recently published an orientation-based

constraint which is independent from excitation and raw measurement data Laidig et al. (2019). One drawback of those methods is that they require specific joint kinematics as well as known joint axes and sensor-to-segment orientations. If these requirements are met, these methods can produce accurate results. The methods for one and two-dimensional joints exploit the limited degrees of freedom to formulate kinematic constraints.

For three-dimensional joints, Kok et al. (2014) and Taetz et al. (2016) exploit positional relationships and use those to estimate the true relative orientations of connected bodies. However, they still rely on magnetometers for the initialisation. In Wenk and Frese (2015) an approach is published which uses an EKF to estimate the orientation of two bodies connected by a ball joint. It is based on the knowledge of the relative position of the IMUs with respect to the joint center. These approaches for three-dimensional joints have in common that they rely on positional constraints and relationships or rely on raw measurement data for heading estimation.

In this paper, we propose a magnetometer-free method for relative motion tracking of kinematic chains with arbitrary joints that have limited range of motion (ROM). The method will be applicable to any rotational (1D, 2D and 3D) joints with ROM constraints, is only based on orientations and does not need known positional relationships or known joint axes. It further does not need any knowledge on initial orientation and yields a long-term stable estimation even under realistic conditions. The proposed method shows remarkable similarities to a Moving Horizon Estimation (MHE) and can be formulated as such. However, we derive and formulate the method in a more intuitive way.

## 2. KINEMATICS AND GENERIC JOINT MODEL

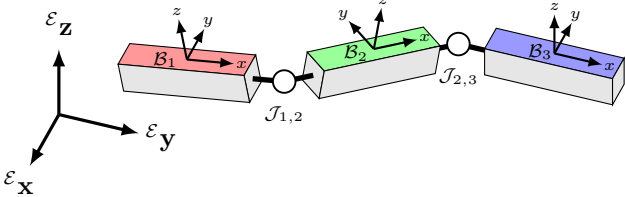


Fig. 1. Model of a kinematic chain consisting of bodies  $\mathcal{B}$  connected by joints  $\mathcal{J}$

Consider a system of  $N$  rigid bodies connected by  $N - 1$  joints. Rigid bodies are denoted  $\mathcal{B}_i$ ,  $i \in [1, N]$ . The joint connecting the bodies  $\mathcal{B}_i$  and  $\mathcal{B}_{i+1}$  is denoted as  $\mathcal{J}_{i,i+1}$ . Joints only connect two adjacent bodies to form a kinematic pair. An example for this system is shown in Fig. 1. We only consider rotational joints which allow for relative rotations of the two connected bodies with respect to each other. The orientation of each segment with respect to a reference frame is expressed by a quaternion. The global reference frame common for all bodies is denoted by  $\mathcal{E}$ . The orientation of  $\mathcal{B}_i$  with respect to  $\mathcal{E}$  is then denoted as  ${}^{\mathcal{E}}\mathbf{q}$ . The relative orientation  ${}^{\mathcal{B}_i}\mathbf{q}$  of two adjacent bodies  $\mathcal{B}_i$  and  $\mathcal{B}_{i+1}$  describes the orientation of  $\mathcal{B}_{i+1}$  with respect to the coordinate system of  $\mathcal{B}_i$  and can

be determined from the orientations of both bodies in the common reference frame:

$${}^{\mathcal{B}_i}\mathbf{q} = {}^{\mathcal{B}_i}\mathbf{q}^{-1} \otimes {}^{\mathcal{B}_{i+1}}\mathbf{q}. \quad (1)$$

The relative orientation of the two bodies is caused by the rotation around the joint connecting the two bodies. Depending on the mechanical model of a joint, it can either allow free relative rotation or the space of possible relative orientations is restricted. These restrictions can either be due to limited degrees of freedom (one- and two-dimensional joints), due to limitations of the range of motion or both. If the joint  $\mathcal{J}_{i+1,i}$  has restrictions of degrees of freedom or range of motion, the set of possible relative orientations  ${}^{\mathcal{B}_i}\mathbf{q}$  for the joint is limited to a subset of all possible orientations  $\mathbb{H}$ . This subset is denoted as  $\mathbb{P}$  with

$$\mathbb{P} \subseteq \mathbb{H}. \quad (2)$$

To describe the relative orientation of the two connected bodies, we propose a generic joint model based on the concept of *joint axes*. We model the rotation  ${}^{\mathcal{B}_i}\mathbf{q}$  from  $\mathcal{B}_i$  to  $\mathcal{B}_{i+1}$  as consecutive rotations around the joint axes  $\mathbf{j}_p \in \mathbb{R}^3$ ,  $\|\mathbf{j}_p\| = 1$ , by the *joint angles*  $\varphi_p \in \mathbb{R}$ ,  $p \in [1 \dots 3]$ . The restriction of the relative orientation is modeled as a limitation of the range of motion of one or more joint angles with

$$\varphi_p \in \{\varphi \in \mathbb{R} \mid \varphi_{p,\min} \leq \varphi \leq \varphi_{p,\max}\}. \quad (3)$$

The relative orientation is then modeled as consecutive rotations around the joint axes

$${}^{\mathcal{B}_i}\mathbf{q} = Q(\varphi_1, \mathbf{j}_1) \otimes Q(\varphi_2, \mathbf{j}_2) \otimes Q(\varphi_3, \mathbf{j}_3) = \prod_{p=1}^3 Q(\varphi_p, \mathbf{j}_p). \quad (4)$$

The operator  $Q(\alpha, \mathbf{j})$  returns the quaternion describing the rotation of  $\alpha$  around the axis  $\mathbf{j}$ . The operator  $\otimes$  denotes quaternion multiplication.

This model can describe most types of rotational joints. For one- and two-dimensional joints, the predefined joint axes are chosen as  $\mathbf{j}_p$ . For the remaining degrees of freedom, linearly independent axes are chosen with a fixed value for the corresponding joint angles, i.e.  $\varphi_{p,\min} = \varphi_{p,\max}$ . This limits the relative orientation to rotations around the predefined joint axes of those joints. For three-dimensional joints, the joint axes  $\mathbf{j}_p$  are chosen either as the predefined axes if the joint is a serial composition of hinge joints (see Fig. 2 left) or can be chosen freely for joints with no distinct axes (see Fig. 2 right). The set which contains all orientations described by the model (4) is denoted as  $\mathbb{P}_M$  and is defined as

$$\mathbb{P}_M := \left\{ \mathbf{q} \in \mathbb{H} \mid \mathbf{q} = \prod_{p=1}^3 Q(\varphi_p, \mathbf{j}_p), \varphi_p \in [\varphi_{p,\min}, \varphi_{p,\max}] \right\}. \quad (5)$$

The range of each joint angle follows from the mechanical model of the joint or from identification experiments.

Without loss of generality we will focus on the description of joints with three degrees of freedom since they describe the most general case. In Fig. 2 two examples for joints with three degrees of freedom are shown. For the left example (a), the joint axes  $\mathbf{j}_p$  and the limits for the joint angles  $\varphi_p$  can be directly be extracted from the mechanical model. If we apply the model from (4) to this, we can see that  $\mathbb{P}_M$  perfectly describes the set of possible relative orientations, therefore  $\mathbb{P} = \mathbb{P}_M$ . For the three-dimensional ball joint (b) any triplet of linearly independent joint axes

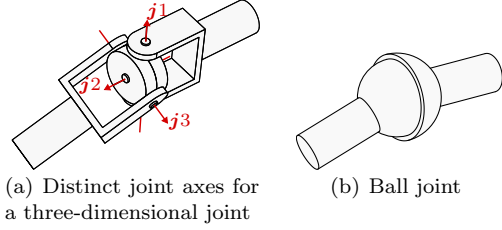


Fig. 2. Two joints with  $\text{dof}(\mathcal{J}) = 3$

can be chosen for the joint model. If we choose three axes  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$  that form an orthonormal base of a right-handed coordinate system, we describe the joint by using Euler angles. This is a common approach and is simpler than choosing non-orthogonal axes. The corresponding Euler angles are denoted as  $\alpha, \beta, \gamma \in \mathbb{R}$  and since the joint has range of motion constraints

$$\alpha \in [\alpha_{\min}, \alpha_{\max}], \beta \in [\beta_{\min}, \beta_{\max}], \gamma \in [\gamma_{\min}, \gamma_{\max}]. \quad (6)$$

If we apply the joint model to this, with  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$  being the joint axes  $\mathbf{j}_p$  and  $\alpha, \beta, \gamma$  the joint angles  $\varphi_p$ , the set described by (5) is a conservative approximation of  $P$  since the joint model assumes fixed ranges for all joint angles. This creates a cuboid subspace in the space created by the three joint angles. For the ball joint however, not the complete subspace can be reached. Therefore, for this joint (and all joints in general)

$$P \subseteq P_M \subseteq \mathbb{H}. \quad (7)$$

Describing each joint with a set of orthogonal axes and Euler angles can be easily applied to most types of rotational joints which have no distinct joint axes.

In the following derivation we only focus on a single kinematic pair of a kinematic chain with  $N$  bodies. The proposed method can then be applied to each joint which fulfills the assumptions to estimate the orientations of the complete kinematic chain. Each joint estimation is independent from the other joints in the chain.

### 3. INERTIAL STATE ESTIMATION

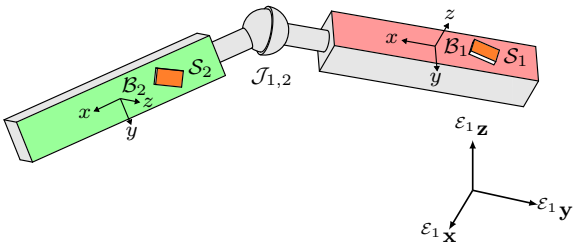


Fig. 3. Kinematic model of two adjacent bodies  $\mathcal{B}_1$  and  $\mathcal{B}_2$  connected by the joint  $\mathcal{J}_{1,2}$  and the reference frame  $\mathcal{E}_1$

Consider two bodies  $\mathcal{B}_1$  and  $\mathcal{B}_2$  connected by the three-dimensional joint  $\mathcal{J}_{1,2}$  with range of motion constraints. To estimate the orientations of the bodies with respect to a reference frame, two inertial sensors  $\mathcal{S}_1$  and  $\mathcal{S}_2$  are placed on the bodies in a known and fixed orientation (see Fig. 3). The sensors measure the angular rates  $\boldsymbol{\omega}_1(t)$  and  $\boldsymbol{\omega}_2(t)$  as well as the accelerations  $\mathbf{a}_1(t)$  and  $\mathbf{a}_2(t)$  in local coordinates of the sensors with a fixed sampling time  $T_s$ . For each segment, 6D quaternion-based sensor

fusion is performed for example according to the algorithm described in Seel and Ruppig (2017), to obtain the orientation of each segment with respect to a reference frame, i.e.  ${}^{\mathcal{B}_1}\mathbf{q}(t)$  and  ${}^{\mathcal{B}_2}\mathbf{q}(t)$ . However, due to the fact that no magnetometer is used and the resulting lack of heading information, the absolute heading of each segment at the beginning of the measurement is arbitrary and only dependent on the initial conditions of the sensor fusion algorithm. This can be modeled as if the orientations of the bodies are estimated in different reference frames  $\mathcal{E}_1$  and  $\mathcal{E}_2$ . Since only the heading of each segment is unknown, the difference between the two reference frames  $\mathcal{E}_1$  and  $\mathcal{E}_2$  can be described by a rotation around the global vertical axis (see Fig. 4). The angle of this rotation is denoted as  $\delta$  and is called *heading offset* Laidig et al. (2017). This rotation is described by the quaternion

$${}^{\mathcal{E}_2}\mathbf{q}(\delta) = [\cos(\frac{\delta}{2}) \ 0 \ 0 \ \sin(\frac{\delta}{2})]^T. \quad (8)$$

With the two estimated orientations  ${}^{\mathcal{B}_1}\mathbf{q}(t)$  and  ${}^{\mathcal{B}_2}\mathbf{q}(t)$

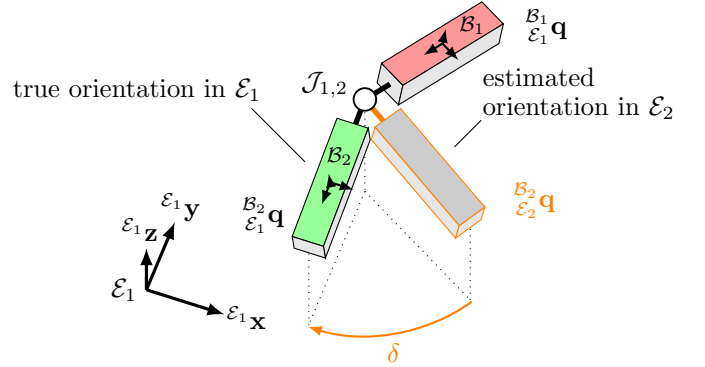


Fig. 4. Difference between the two reference frames  $\mathcal{E}_1$  and  $\mathcal{E}_2$

and a known value of the heading offset  $\delta$  we can determine the relative orientation of the two bodies by

$${}^{\mathcal{B}_2}\mathbf{q}(t) = {}^{\mathcal{B}_1}\mathbf{q}(t)^{-1} \otimes {}^{\mathcal{E}_2}\mathbf{q}(\delta) \otimes {}^{\mathcal{B}_2}\mathbf{q}(t). \quad (9)$$

Due to bias in the gyroscope measurements and the resulting drift around the global vertical axis, the angle  $\delta$  is not constant and can be approximated as

$$\delta(t) \approx \underbrace{\frac{\partial \delta}{\partial t}}_{\text{heading drift}} \cdot t + \underbrace{\delta(t=0)}_{\text{initial offset } \delta_0}. \quad (10)$$

$\frac{\partial \delta}{\partial t}$  is a scalar which is slowly changing due to bias instability and non-uniformity of the bias of each axis. For small time windows the heading offset  $\delta(t)$  can be approximated constant, since the drift is typically small with magnitudes of  $\frac{\partial \delta}{\partial t} < 0.5 \frac{\circ}{s}$ . Note that we do not make any other assumptions on the course of  $\delta(t)$ .

### 4. CONSTRAINT

Consider two bodies  $\mathcal{B}_1$  and  $\mathcal{B}_2$  connected by the joint  $\mathcal{J}$  with the most general case of three degrees of freedom. The joint has mechanical restrictions which limits the set of possible relative orientations to  $P \subset \mathbb{H}$ . The kinematics of the joint are described by an Euler angle convention with the orthonormal axes  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$  and the corresponding angles  $\alpha$ ,  $\beta$ ,  $\gamma$ , which are restricted according to (6).

The set of relative orientations described by this model is  $P_M$  with  $P \subseteq P_M$ . At any given point in time  $t_k$ , the orientations of the two segments estimated by the 6D sensor fusion are  ${}_{\mathcal{E}_1}^{B_1}\mathbf{q}(t_k)$  and  ${}_{\mathcal{E}_2}^{B_2}\mathbf{q}(t_k)$ . The heading offset at that time instant is  $\delta(t_k)$ . Let  $\hat{\delta}$  be an estimate of  $\delta(t_k)$ , then an estimate  ${}_{B_1}^{B_2}\hat{\mathbf{q}}$  of the relative orientation can be obtained by

$${}_{B_1}^{B_2}\hat{\mathbf{q}}(t_k, \hat{\delta}) := {}_{\mathcal{E}_1}^{B_1}\mathbf{q}(t_k)^{-1} \otimes {}_{\mathcal{E}_1}^{\mathcal{E}_2}\hat{\mathbf{q}}(\hat{\delta}) \otimes {}_{\mathcal{E}_2}^{B_2}\mathbf{q}(t_k). \quad (11)$$

Let  $E_{abc} : \mathbb{H} \mapsto \mathbb{R}^3$  be an operator that maps any given quaternion  $\mathbf{q}$  to a triplet of Euler angles  $(\alpha, \beta, \gamma)$  according to the intrinsic Euler angles convention  $a-b'-c''$  Diebel (2006). To check whether  ${}_{B_1}^{B_2}\hat{\mathbf{q}}(t_k, \hat{\delta})$  is an element of  $P_M$  at a given time instant  $t_k$  and a given estimate  $\hat{\delta}$ , it has to be decomposed into the Euler angles  $\hat{\alpha}$ ,  $\hat{\beta}$  and  $\hat{\gamma}$  corresponding to the Euler angle convention  $a-b'-c''$

$$(\hat{\alpha}, \hat{\beta}, \hat{\gamma}) = E_{abc} \left( {}_{B_1}^{B_2}\hat{\mathbf{q}}(t_k, \hat{\delta}) \right). \quad (12)$$

If  $\hat{\alpha}$ ,  $\hat{\beta}$  and  $\hat{\gamma}$  are within the specified ranges according to (6), then it is a valid relative orientation  ${}_{B_1}^{B_2}\hat{\mathbf{q}}(t_k, \hat{\delta}) \in P_M$  according to the model. We define a function  $e_k(\hat{\delta}) : \mathbb{R} \mapsto \mathbb{N}$  that assigns a binary value 0 or 1 to an estimate  $\hat{\delta}$  at a given time instant  $t_k$  with

$$e_k(\hat{\delta}) := \begin{cases} 0, & \text{if } {}_{B_1}^{B_2}\hat{\mathbf{q}}(t_k, \hat{\delta}) \in P_M \\ 1, & \text{otherwise.} \end{cases} \quad (13)$$

## 5. ESTIMATION PRINCIPLE

The basic idea of the estimation of  $\delta(t)$  is, that at each time instant  $t_k$ , only a subset of possible values of  $\hat{\delta}$  produces a relative orientation  ${}_{B_1}^{B_2}\hat{\mathbf{q}}(t_k, \hat{\delta})$  according to (1), that lies within the set of possible relative orientations  $P_M$ . With the assumption, that  $\delta(t)$  can be approximated constant over a small time window and the assumption of sufficiently rich motion, only one value of  $\hat{\delta}$  produces valid relative orientations for all time instants of that time window, i.e. does not violate the constraint. We therefore use a window-based approach to find a good estimate of  $\delta(t)$  for a given time interval  $T_w$ .

At regular time intervals  $T_{\text{est}} \geq T_s$ , an estimation of  $\delta(t)$  is performed at the time instants  $t_w$ , which are denoted with an index  $w \in \mathbb{N}^+$  and are defined as

$$t_w := wT_{\text{est}}. \quad (14)$$

Each time window consists of  $N_w$  samples taken at the sampling interval  $T_s$  at the sampling instants  $t_k$ . The number of samples within a window is denoted as  $N_w$ . The time instants corresponding to the time window  $w$  are given as

$$t_k = t_w + (m - N_w)T_s, \quad m \in [1 \dots N_w]. \quad (15)$$

We choose  $T_{\text{est}} < T_w$  to create overlapping time windows. The time window definition only uses samples before the current estimation time instant, making the method real-time capable.

We assume that for sufficiently small time windows the heading offset  $\delta(t)$  can be approximated as constant. The estimate for the heading offset during a given time window  $w$  is denoted by  $\hat{\delta}_w$ . Following this assumption and the model of  $\delta(t)$  from (10), we assume that from

one time window to the next one the value  $\hat{\delta}_w$  does not change rapidly and is close to the previous value  $\hat{\delta}_{w-1}$ . We formulate a cost function  $c_w(\hat{\delta}) : \mathbb{R} \mapsto \mathbb{R}$  that for a given time window  $w$  assigns a cost to a value of  $\hat{\delta}$

$$c_w(\hat{\delta}) := \frac{N_w}{\pi} |\hat{\delta} - \hat{\delta}_{w-1}| + \sum_{m=1}^{N_w} e_k(\hat{\delta}), \quad (16)$$

with  $k = wT_{\text{est}} + (m - N_w)$ .

The first term penalizes the distance to the previous estimate  $\hat{\delta}_{w-1}$ . This ensures that the new estimate  $\hat{\delta}_w$  is close to the previous estimate. The scaling factor  $\frac{N_w}{\pi}$  scales the cost that a distance of  $\pi$  is equal to the maximum cost of the second term. We only penalize the distance and do not make assumptions on the direction of change. The second term is a measure of how well the estimate  $\hat{\delta}$  fulfills the kinematic constraint. The more valid relative orientations it produces over the course of the time window, the smaller the second term gets. (16) is then used to find an estimate  $\hat{\delta}_w$  for a time window that minimizes the cost over that window by the help of any optimization method

$$\hat{\delta}_w := \arg \min_{\hat{\delta}} c_w(\hat{\delta}). \quad (17)$$

Note that the proposed method can be interpreted as a moving-horizon estimation approach for a dynamical system with one state  $\delta$ , which has an uncertain but small time derivative, and a highly non-linear output (13), which we know to be zero up to small inaccuracies. The cost function (16) combines penalizes outputs different from this virtual zero measurements as well as state values that disagree with the uncertain dynamics.

For any time instant  $t_k$  the value of the estimated heading offset  $\hat{\delta}(t_k)$  can be determined with

$$\hat{\delta}(t_k) = \hat{\delta}_{\tilde{w}} \quad \text{with} \quad \tilde{w} = \left\lfloor \frac{k}{N_w} \right\rfloor \in \mathbb{N}^+. \quad (18)$$

In Fig. 5 three examples for possible cost functions  $c_w(\hat{\delta})$  over  $\hat{\delta} \in [0, 2\pi]$  are shown. In the left graph, the constraint part of the cost function dominates. In the right graph, the distance cost to the previous estimate has more impact and ensures a non-diverging estimate even if the constraint minimum is less distinct. With an estimate

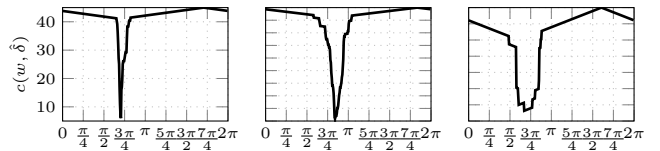


Fig. 5. Examples for cost functions  $c_w(\hat{\delta})$  over  $\hat{\delta} \in [0, 2\pi]$

$\hat{\delta}(t)$ , both estimated orientations  ${}_{\mathcal{E}_1}^{B_1}\mathbf{q}(t)$  and  ${}_{\mathcal{E}_2}^{B_2}\mathbf{q}(t)$  can be transformed into a common reference frame. This allows us to calculate the relative orientation, joint angles and positional relationships between the connected bodies or to transform all body orientations of a kinematic chain into one common reference frame.

## 6. EXPERIMENTAL VALIDATION

The proposed method is validated experimentally with the use of a mechanical three-dimensional joint as a test object. As validation method an optical motion capture system is used to measure the relative orientation with very high precision. We investigate different motion patterns and speeds to evaluate the robustness of the method.

### 6.1 Setup

The test object is 3D printed (see Fig. 6) with a well-defined three-dimensional joint connecting the two bodies. The range of motion of the joint angles are based on the range of motion of the Carpometacarpal joint of the thumb Salchow-Hömmen et al. (2019) of the author. The joint is

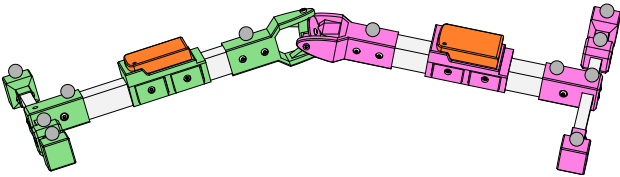


Fig. 6. Mechanical model used for experimental evaluation. The two bodies are connected by a well-defined 3D joint. On each segment IR-reflective markers are attached for optical motion capture.

modeled by a  $z$ - $x'$ - $y''$  Euler angles convention with the angle ranges

$$\alpha \in [-20^\circ, 20^\circ], \quad \beta \in [-15^\circ, 15^\circ], \quad \gamma \in [-40^\circ, 40^\circ]. \quad (19)$$

The joint axes  $\mathbf{j}_1$ ,  $\mathbf{j}_2$  and  $\mathbf{j}_3$  are known and coincide with the axes of the  $z$ - $x'$ - $y''$  Euler rotation sequence. On each segment, an IMU is placed in a known and constant orientation. The IMUs measure the angular velocity and the acceleration of each body at a rate of 75 Hz and 6D sensor fusion is performed using the algorithm presented in Seel and Ruppig (2017).

### 6.2 Optical reference

As a ground truth for the relative orientation of the two bodies an optical motion capture system (MoCap) is used. It consists of ten *Optitrack Flex 13* cameras, attached in a rectangular pattern at the top of the measurement space. On the bodies 11 (5+6) IR-reflective markers are attached in a well-defined and known pattern (see Fig. 6). The MoCap system determines the 3D positions of the markers at a rate of 120 Hz with a mean position error of less than 0.3 mm. With the marker positions, each segment's orientation can be calculated at each sample time instant in a common reference frame and with (1) the true relative orientation of both bodies can be determined.

### 6.3 Validation metrics

Let  ${}_{\mathcal{B}_1}^{\mathcal{B}_2} \hat{\mathbf{q}}(t)$  be the estimated relative orientation of the two bodies determined by the algorithm described above. Let  ${}_{\mathcal{B}_1}^{\mathcal{B}_2} \mathbf{q}(t)$  be the true relative orientation determined by the MoCap system. Both timeseries are in a common timeframe. We define the *orientation error*  $\epsilon(t)$  to be

the error angle between the estimated and true relative orientation, i.e.

$$\epsilon(t) := \left| A \left( {}_{\mathcal{B}_1}^{\mathcal{B}_2} \mathbf{q}(t)^{-1} \otimes {}_{\mathcal{B}_1}^{\mathcal{B}_2} \hat{\mathbf{q}}(t) \right) \right|, \quad (20)$$

with  $A(\mathbf{q})$  being the operator that extracts the angle of the corresponding quaternion  $\mathbf{q}$ . To quantify the method's ability to estimate the heading offset  $\delta(t)$  we introduce the error  $\epsilon_\delta(t)$ , which is defined as the difference between the true and estimated value of  $\delta(t)$

$$\epsilon_\delta := |\delta(t) - \hat{\delta}(t)|. \quad (21)$$

The error  $\epsilon_\delta(t)$  is the heading component of  $\epsilon(t)$  after being transformed into the global reference frame. To quantify the overall performance over the course of an experiment, we use the metrics  $\epsilon_{\text{RMS}}$  and  $\epsilon_{\delta, \text{RMS}}$ , which are the RMSE of their corresponding error metric.

### 6.4 Conducted experiments

Multiple short-term and long-term experiments have been conducted to test the initial estimation as well as the long-term stability. At the beginning of each experiment, the test object is resting in a random orientation on a table. It is then picked up and rotated and translated within the measurement space of the MoCap system. In all experiments the magnitude and frequency of excitation have been varied. A list of the experiments with their duration is given in Table 1. For the estimation we use

Table 1. Performed experiments

Experiment	Duration	Remark
E.01	64 s	random starting orientation
E.02	62 s	random starting orientation
E.03	64 s	random starting orientation
E.04	301 s	fast movement
E.05	212 s	slow movement
E.06	305 s	mixed movement with pauses
E.07	418 s	very long measurement with mixed movement

a window time  $T_w = 8$  s and an estimation time  $T_{\text{est}} = 1$  s.

### 6.5 Results

The method has to fulfill two objectives: converge towards the initial heading offset  $\delta_0$  at the beginning of the estimation and therefore the initial relative orientation as well as track the course of  $\delta(t)$  over an arbitrary length of the experiment for long-term stability. In Fig. 7 the error  $\epsilon(t)$  for all experiments is shown for the first 10 seconds as well as for the complete experiments. Despite initial errors  $> 20^\circ$ , in all cases the converges below  $5^\circ$  within 5 s and drops below  $4^\circ$  after  $t > 60$  s. The mean RMS for all experiments is  $\epsilon_{\text{RMS}} = 1.9^\circ$  with a heading tracking error of  $\epsilon_{\delta, \text{RMS}} = 0.8^\circ$ . This shows that the method can estimate the true relative orientation very fast at the beginning of each estimation, which eradicates the need for predefined initial poses. The method is also able to track the relative orientation with a high accuracy even for long experiments and without external heading correction.

In Fig. 8 the estimation of  $\delta(t)$  for one of the experiments is shown. The estimate follows the reference accurately with  $\epsilon_{\delta, \text{RMS}} = 0.9^\circ$ . During the interval  $t \in [164 \text{ s}, 184 \text{ s}]$  the test

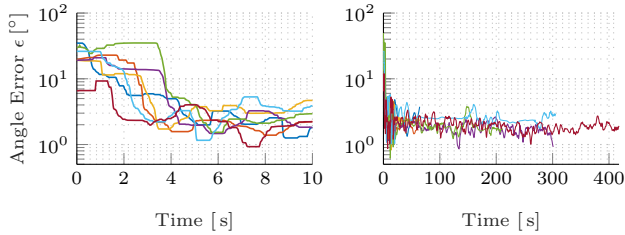


Fig. 7. Error  $\epsilon(t)$  for all experiments. Left: initial convergence during the first 10 seconds. Right: Error of the complete experiments.

object is lying down with no excitation. During that phase the first term of (16) ensures that the estimate does not diverge and stays close to the true value of  $\delta(t)$ .

In Fig. 9 the error  $\epsilon(t)$  is shown for the proposed method (KC) and, as a benchmark, 6D sensor fusion without constraints and only initial heading correction and bias compensation (6D) Salchow-Hömmen et al. (2019) as well as conventional 9D sensor fusion with magnetometer correction (9D). The proposed method performs best, with  $\epsilon_{\text{RMS}} = 2.2^\circ$ . The 6D method produces adequate results at the beginning of the estimation but due to drift the relative orientation diverges, leading to a maximum error of  $34^\circ$ . Conventional 9D sensor fusion produces temporarily accurate results, but due to magnetic disturbances it has a RMS of  $\epsilon_{\text{RMS}} = 6.8^\circ$  with a maximum error of  $18^\circ$ . The mean results for all experiments are given in Table

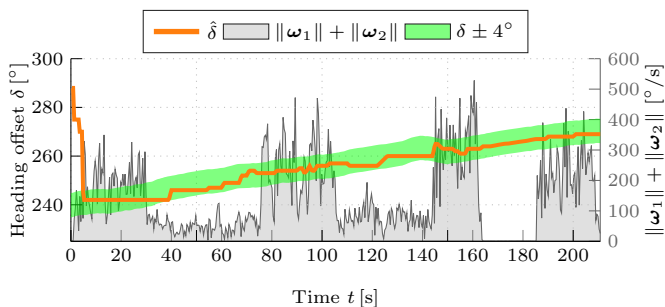


Fig. 8. Estimate  $\hat{\delta}(t)$  and reference value  $\delta(t)$  for experiment E\_05 as well as norm of gyroscope measurements.

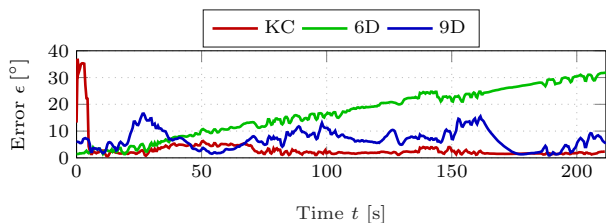


Fig. 9. Errors  $\epsilon(t)$  for the three evaluated methods for the experiment E\_05.

2. Additionally the results for the conventional 6D and 9D sensor fusion methods are presented. The values in parentheses indicate the maximum errors  $\max(\epsilon(t))$  and  $\max(\epsilon_\delta(t))$  of all experiments. The proposed method produces the best results with mean orientation errors of  $2.1^\circ$

Table 2. Mean (Max) errors for all experiments

Error	KC	6D	9D
$\epsilon_{\text{RMS}}$	<b>2.1</b> <sup>(4°)</sup>	6.1 <sup>(34°)</sup>	6.3 <sup>(21°)</sup>
$\epsilon_{\delta, \text{RMS}}$	<b>0.8</b> <sup>(2°)</sup>	5.4 <sup>(32°)</sup>	5.9 <sup>(20°)</sup>

and maximum orientation errors of  $4^\circ$ . The objective of the method to track the heading offset can be accomplished with a mean error of less than  $1^\circ$  for all experiments. The two conventional methods produce worse results with mean errors of approx.  $6^\circ$  and maximum errors larger than  $20^\circ$ .

## 7. CONCLUSION

We proposed a new method for magnetometer-free inertial motion tracking for arbitrary joints with limited range of motion. The method exploits ROM constraints and the limited set of possible relative orientations to estimate the relative heading of two connected bodies using a window-based approach. To apply the method to different joint geometries we proposed a generic joint model for rotational joints which can sufficiently approximate joints with no distinct joint axes.

The method enables magnetometer-free real-time tracking in real-world indoor environments. We showed that the method yields an accurate long-term stable estimate for a case study derived from a real physiological joint. Compared to other methods Salchow-Hömmen et al. (2019) that also omit the magnetometer readings, the method does not rely on known initial poses and converges towards the true relative orientation within seconds. We showed for the evaluated test object that the method yields better results than conventional 9D motion tracking and is long-term stable in contrast to 6D motion tracking methods.

However, compared to different orientation-based constraints that exploit the limited degrees of freedom of one- and two-dimensional joints Laidig et al. (2017, 2019), the method needs recurrent excitation. Furthermore, the method only works for joints where at least one of the joint angles has a moderate range of motion restriction. It has to be ensured to find the Euler angle convention which produces the smallest ranges for the joint angles.

Future work aims at overcoming the set limitations, i.e. investigating the sufficiency of rich motion, the magnitude of the angular restrictions as well as validating the method in less rigid, biological joints.

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## REFERENCES

- Cooper, G., Sheret, I., McMillian, L., Siliverdis, K., Sha, N., Hodgins, D., Kenney, L., and Howard, D. (2009). Inertial sensor-based knee flexion/extension angle estimation. *Journal of Biomechanics*, 42(16), 2678–2685. doi:10.1016/j.jbiomech.2009.08.004.
- de Vries, W.H.K., Veeger, H.E.J., Baten, C.T.M., and van der Helm, F.C.T. (2009). Magnetic distortion in motion labs, implications for validating inertial magnetic

- sensors. *Gait & Posture*, 29(4), 535–541. doi:10.1016/j.gaitpost.2008.12.004.
- Diebel, J. (2006). Representing attitude : Euler angles, unit quaternions, and rotation vectors.
- Fong, D.T.P. and Chan, Y.Y. (2010). The use of wearable inertial motion sensors in human lower limb biomechanics studies: A systematic review. *Sensors*, 10(12), 11556–11565. doi:10.3390/s101211556.
- Grand, E.L. and Thrun, S. (2012). 3-axis magnetic field mapping and fusion for indoor localization. In *2012 IEEE International Conference on Multisensor Fusion and Integration for Intelligent Systems (MFI)*, 358–364. doi:10.1109/MFI.2012.6343024.
- Kok, M., Hol, J.D., and Schön, T.B. (2014). An optimization-based approach to human body motion capture using inertial sensors. *IFAC Proceedings Volumes*, 47(3), 79–85. doi:10.3182/20140824-6-ZA-1003.02252.
- Laidig, D., Schauer, T., and Seel, T. (2017). Exploiting kinematic constraints to compensate magnetic disturbances when calculating joint angles of approximate hinge joints from orientation estimates of inertial sensors. In *2017 International Conference on Rehabilitation Robotics (ICORR)*, 971–976. doi:10.1109/ICORR.2017.8009375.
- Laidig, D., Lehmann, D., Bégin, M.A., and Seel, T. (2019). Magnetometer-free realtime inertial motion tracking by exploitation of kinematic constraints in 2-dof joints. *2019 41st Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC)*, 1233–1238.
- Miller, N., Jenkins, O.C., Kallmann, M., and Mataric, M.J. (2004). Motion capture from inertial sensing for untethered humanoid teleoperation. In *4th IEEE/RAS International Conference on Humanoid Robots, 2004.*, volume 2, 547–565 Vol. 2. doi:10.1109/ICHR.2004.1442670.
- Salchow-Hömmen, C., Callies, L., Laidig, D., Valtin, M., Schauer, T., and Seel, T. (2019). A tangible solution for hand motion tracking in clinical applications. *Sensors*, 19(1), 208. doi:10.3390/s19010208.
- Seel, T., Raisch, J., and Schauer, T. (2014). IMU-based joint angle measurement for gait analysis. *Sensors*, 14(4), 6891–6909. doi:10.3390/s140406891.
- Seel, T. and Ruppig, S. (2017). Eliminating the effect of magnetic disturbances on the inclination estimates of inertial sensors. *IFAC-PapersOnLine*, 50(1), 8798–8803. doi:10.1016/j.ifacol.2017.08.1534.
- Shu, Y., Bo, C., Shen, G., Zhao, C., Li, L., and Zhao, F. (2015). Magicol: Indoor localization using pervasive magnetic field and opportunistic WiFi sensing. *IEEE Journal on Selected Areas in Communications*, 33(7), 1443–1457. doi:10.1109/JSAC.2015.2430274.
- Subbu, K.P., Gozick, B., and Dantu, R. (2013). LocateMe: Magnetic-fields-based indoor localization using smartphones. *ACM Trans. Intell. Syst. Technol.*, 4(4), 73:1–73:27. doi:10.1145/2508037.2508054.
- Taetz, B., Bleser, G., and Miezal, M. (2016). Towards self-calibrating inertial body motion capture. In *2016 19th International Conference on Information Fusion (FUSION)*, 1751–1759.
- Wenk, F. and Frese, U. (2015). Posture from motion. In *2015 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, 280–285. doi:10.1109/IROS.2015.7353386.
- Wong, C., Zhang, Z., Lo, B., and Yang, G. (2015). Wearable sensing for solid biomechanics: A review. *IEEE Sensors Journal*, 15(5), 2747–2760. doi:10.1109/JSEN.2015.2393883.