Distributed Conditional Cooperation Model Predictive Control of Interconnected Microgrids ⋆

Ajay Kumar Sampathirao [a], Steffen Hofmann [b], Jörg Raisch [b], Christian A. Hans [b]

[a] *Enervalis, Greenville Campus, Centrum-Zuid 1111, 3530 Houthalen-Helchteren, Belgium*

[b] *Technische Universität Berlin, Einsteinufer 11, 10587 Berlin, Germany*

Abstract

In this paper, we propose a model predictive control based operation strategy that allows for power exchange between interconnected microgrids (MGs). Particularly, the proposed conditional cooperation approach ensures that each MG in the network benefits from power exchange with others. This is realised by including a condition which is based on the islanded operation cost and preserves the self-interest of each MG. The overall model predictive control problem is posed as a mixed-integer quadratically-constrained program and solved using a distributed algorithm that iteratively updates continuous and integer variables. For this algorithm, termination, feasibility and computational properties are discussed. The performance of the proposed strategy and the computational benefits of the distributed algorithm are highlighted in an illustrative case study.

1 Introduction

Recent advances in renewable energies and growing concerns about environmental impacts of fossil-fuelled power plants led to a worldwide increase of renewable energy sources (RES) such as photovoltaic generators and wind turbines [17]. RES are often small-scale distributed units (DUs), characterised by intermittent power output and geographical proximity to consumers. At this juncture, the microgrid (MG) concept is a promising approach to facilitate the integration of a large number of RES. An MG refers to a self-contained system with local demand, generators and storage units. MGs can operate connected to or isolated from the power network, i.e., islanded, [22]. For both modes, operation control serves to coordinate the DUs on a timescale of minutes.

Connecting MGs and facilitating power exchange between them can increase flexibility compared to islanded operation [20]. The interconnection allows the MGs to benefit from different infeed pattern of geographically distributed and technological diverse RES. In this paper, we develop a strategy that manages power exchange while preserving the self-interest of each MG in a network. The derived conditional cooperation control scheme is composed of two parts: (i) estimation of a condition for each MG that reflects its self-interest; and (ii) determination of power exchange between the MGs in the network such that the condition from (i) is satisfied for each MG. The scheme is based on model predictive control (MPC) which is widely adopted for the operation of MGs as it allows to explicitly consider unit limits as well as economic objectives [8, 13].

Coordinating multiple MGs in a network has been widely discussed in literature, e.g., in [13, 18, 26] and the references therein. All of the aforementioned publications highlight the benefits of coordinated power transfers in interconnected MGs. However, they focus on minimising the combined operating cost which does not always secure the self-interest of each individual MG. This drawback has been dealt with in [15,19]. Unfortunately, both approaches require a central unit and are therefore not suitable for a fully distributed computation.

Distributed MPC that considers cooperation of individual agents while prioritising local goals has been studied, e.g., in [23,24]. Venkat *et al.* [24] proposed a cooperation-based distributed MPC approach, which iteratively exchanges information between the agents to find feasible control actions. Valencia *et al.* [23] formulated a feasible-cooperation MPC as a cooperative game that allows each agent to decide whether it wants to cooperate or not.

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Email addresses: ajay.sampathirao@enervalis.com (Ajay Kumar Sampathirao), hofmann@control.tu-berlin.de (Steffen Hofmann), raisch@control.tu-berlin.de (Jörg Raisch), hans@control.tu-berlin.de (Christian A. Hans).

The main limitation of most approaches is their restriction to convex functions. In the context of MG operation control, this means that the on/off condition of conventional generators cannot be directly implemented. This limitation was addressed in [6], where a hierarchical distributed MPC that includes the switching of conventional units was proposed. The binary variables that represent their on/off condition were relaxed and power flow was decided in a distributed manner using the alternating direction method of multipliers. However, the approach did not preserve the MGs' self-interest.

The main contributions of this paper are as follows. (i) A conditional cooperation MPC problem for the operation of interconnected MGs is posed as a mixed-integer quadratically-constrained program (MIQCP). It includes a condition that guarantees preservation of each MG's self-interest, based on the cost of islanded operation. The MPC problem is solved to determine the power of each DU in each MG as well as power exchange over the grid. (ii) We propose a feasible-decomposition-based algorithm to find a (not necessarily optimal) solution to the aforementioned MIQCP. This algorithm comes with a significantly reduced computational complexity compared to the original MIQCP. We study the properties of the algorithm and provide a criterion that guarantees termination in a finite number of steps. (iii) A fully distributed version of the feasible-decomposition-based algorithm is proposed. Here, the power exchange of the interconnected MGs is determined using a distributed augmented Lagrangian method and the binary variables are updated based on local subproblems. (iv) In an extensive case study, we compare the central solution with the one from the distributed algorithm. Moreover, we examine properties of the algorithm, e.g., convergence, and highlight its applicability in real-world settings.

The remainder of the paper is structured as follows. In Section 2, a microgrid and a network model are introduced. Then, in Section 3 the operation costs are derived and the conditional cooperation MPC problem is posed. In Section 4, an algorithm which can be used to solve the previously defined MPC problem is deduced. Then, in Section 5 the distributed implementation of the algorithm is discussed. Finally, in Section 6 a numerical case study that illustrates the performance of the proposed distributed algorithm is presented.

1.1 *Notation and mathematical preliminaries*

The set of real numbers is denoted by $\mathbb{R}$, the set of nonpositive real numbers by $\mathbb{R}_{\leq 0}$, the set of nonnegative real numbers by $\mathbb{R}_{\geq 0}$, the set of positive real numbers by $\mathbb{R}_{>0}$ and the set of negative real numbers by $\mathbb{R}_{<0}$. The set of positive integers is denoted by $\mathbb{N}$, the set of nonnegative integers by $\mathbb{N}^0$ and the set of the first n positive integers by $\mathbb{N}_n = \{1, 2, \ldots, n\}$. Moreover, we define $\mathbb{N}_{[0,n]} = \{0\} \cup \mathbb{N}_n$. The cardinality of a set $\mathbb{A}$ is $|\mathbb{A}|$. The vector $[x_{v_1}\ x_{v_2}\ \cdots\ x_{v_n}]$ for all $v_i \in V = \{v_1, v_2, \cdots, v_n\} \subset \mathbb{N}$ with $v_i < v_j$ for $i < j$ is denoted by $\mathrm{vec}(x_{v_i})_{v_i \in V}$. We denote the vector $\mathrm{vec}(\mathrm{vec}(x_{v_i u_i})_{u_i \in U})_{v_i \in V}$ by $\mathrm{vec}(x_{v_i u_j})_{v_i \in V, u_j \in U}$. The Cartesian product of sets A and B is $A \times B$. Moreover, $\|\cdot\|$ is the L_2 norm and $|c|$ the absolute value of $c \in \mathbb{R}$.

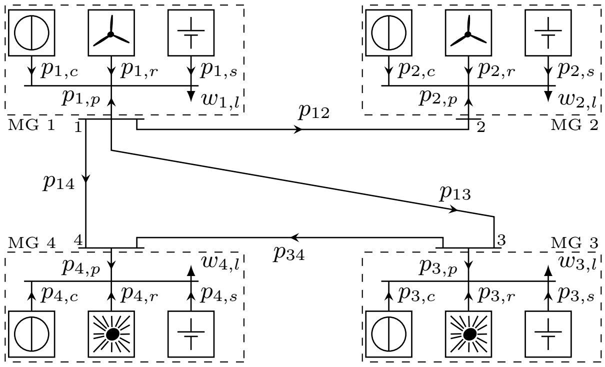

Fig. 1. Topology of four interconnected MGs from [6].

Consider an undirected connected graph $\mathcal{G} = (N, E)$ where $N = \mathbb{N}_n$ is the set of nodes and $E \subseteq [N]^2$ the set of edges linking the nodes. Here, $[N]^2$ denotes the set of all two-element subsets of N. Node j is a neighbour of node i, if there is an edge $\{i, j\} \in E$. The set of neighbours of node j is $N_j = \{i \mid i \in N, \{i, j\} \in E\}$ and the number of neighbours of j is $n_j = |N_j|$.

2 Model of interconnected MGs

In this section, we introduce a control-oriented discrete-time model of a network of interconnected MGs. First, the model of a single MG is presented. Then, the model of the grid connecting the MGs is introduced.

The model is motivated by [6, 9] and considers $n \in \mathbb{N}$ MGs that are connected by an electrical network (see Fig. 1). Each MG_j, $j \in \mathbb{N}_n$, is connected to a bus in the grid via a point of common coupling (PCC) that allows for power exchange with others. Despite this interconnection, each MG is assumed to be capable of operating in islanded mode without power exchange via the PCC.

2.1 *Model of single microgrid*

The mathematical model of a considered MG is composed of unit constraints, energy dynamics and constraints that account for a local power balance. Note that the control scheme derived in this paper can be easily modified to work with alternative MG configurations, such as, MGs with controllable loads or MGs in which renewable infeed cannot be curtailed.

Each MG_j is composed of $g_j \in \mathbb{N}$ DUs and $l_j \in \mathbb{N}$ loads. Each unit $i \in \mathbb{N}_{g_j}$ is labelled $\mathrm{DU}_{j,i}$ such that we can define the index set of conventional generators by $C_j = \{i \mid i \in \mathbb{N}_{g_j}, \mathrm{DU}_{j,i} \text{ is a conventional unit}\}$. Similarly, S_j is the index set of storage units, R_j the index set of RES and $L_j = \{g_j{+}1, \ldots, g_j{+}l_j\} \subset \mathbb{N}$ the index set of loads. The number of conventional, storage and renewable units is $|C_j|$, $|S_j|$ and $|R_j|$ with $g_j = |C_j|{+}|S_j|{+}|R_j|$.

2.1.1 *Renewable energy sources*

RES like photovoltaic generators or wind turbines are assumed to be the predominant energy providers in the MGs. The power from these units is uncertain as their availability depends on intermittent weather conditions, e.g., solar irradiance or wind speed. At time instant $k \in \mathbb{N}^0$, the available power of renewable unit $i \in R_j$ is denoted by $w_{j,i}(k)$. The power infeed $p_{j,i}(k)$ of a renewable unit is limited by the available power $w_{j,i}(k)$, which, in turn, is bounded by the rated power $\overline{p}_{j,i} \in \mathbb{R}_{\geq 0}$, i.e.,

$$0 \leq p_{j,i}(k) \leq w_{j,i}(k) \leq \overline{p}_{j,i}, \quad \forall i \in R_j. \tag{1}$$

2.1.2 *Storage units*

We consider battery-based storage units that compensate fluctuations of RES and loads. Stored energy and power of unit $i \in S_j$ are denoted by $x_{j,i}(k)$ and $p_{j,i}(k)$, respectively. The dynamics depend on whether a storage unit is charging or discharging. We consider a binary variable $\delta_{j,i}(k) \in \{0,1\}$ which is 1 if unit i is charging and 0 if it is discharging. With the limits $\underline{p}_{j,i} \in \mathbb{R}_{<0}$ and $\overline{p}_{j,i} \in \mathbb{R}_{>0}$, the storage power constraints reads

$$\underline{p}_{j,i}\delta_{j,i}(k) \leq p_{j,i}(k) \leq (1-\delta_{j,i}(k))\overline{p}_{j,i}, \quad \forall i \in S_j. \tag{2a}$$

Note that (2a) is $\underline{p}_{j,i} \leq p_{j,i}(k) \leq 0$ for $\delta_{j,i}(k) = 1$ (charging) and $0 \leq p_{j,i}(k) \leq \overline{p}_{j,i}$ for $\delta_{j,i}(k) = 0$ (discharging).

Using the sampling time $T_\mathrm{s} \in \mathbb{R}_{>0}$ and the storage efficiency $\eta_{j,i} \in (0,1]$, the dynamics of storage unit i are

$$\begin{aligned} x_{j,i}(k+1) &= x_{j,i}(k) - \eta_{j,i}\delta_{j,i}(k)T_\mathrm{s}p_{j,i}(k) \\ &\quad - \tfrac{1}{\eta_{j,i}}(1-\delta_{j,i}(k))T_\mathrm{s}p_{j,i}(k), \quad \forall i \in S_j. \end{aligned} \tag{2b}$$

The energy is bounded by $\underline{x}_{j,i} \in \mathbb{R}_{\geq 0}$ and $\overline{x}_{j,i} \in \mathbb{R}_{>0}$,i.e.,

$$\underline{x}_{j,i} \leq x_{j,i}(k) \leq \overline{x}_{j,i}, \quad \forall i \in S_j. \tag{2c}$$

Remark 1 *Note that using* (2b)*, which comprises a multiplication of decision variables $\delta_{j,i}(k)$ and $p_{j,i}(k)$, would result in a nonlinear optimization problem. Fortunately, with the help of an additional real-valued decision variable for each storage unit,* (2b) *can be easily transformed into a set of affine equality and inequality constraints (see, e.g., [14]). Therefore,* (2) *can be used in mixed-integer linear or quadratic optimization problems, e.g., Problems 1 and 4.*

2.1.3 *Conventional generators*

Typically, conventional units are used as a backup in times of low renewable infeed and little stored energy. Let us represent the on/off condition of unit $i \in C_j$ by the binary variable $\delta_{j,i} \in \{0,1\}$ where $\delta_{j,i} = 0$ means that it is disabled and $\delta_{j,i} = 1$ that it is enabled. The power output of conventional generator i is limited by

$$\underline{p}_{j,i}\delta_{j,i}(k) \leq p_{j,i}(k) \leq \overline{p}_{j,i}\delta_{j,i}(k), \quad \forall i \in C_j, \tag{3}$$

with bounds $\underline{p}_{j,i} \in \mathbb{R}_{>0}$ and $\overline{p}_{j,i} \in \mathbb{R}_{>0}$ Note that the above constraint becomes $p_{j,i}(k) = 0$ for $\delta_{j,i}(k) = 0$ and $\underline{p}_{j,i} \leq p_{j,i}(k) \leq \overline{p}_{j,i}$ for $\delta_{j,i}(k) = 1$.

2.1.4 *Loads*

Uncontrollable electric demand is modelled as an uncertain input. It is denoted by $w_{j,i}(k) \in \mathbb{R}_{\leq 0}$ with $i \in L_j$.

2.1.5 *Point of common of coupling (PCC)*

The PCC connects MG_j with the transmission network and thus allows for power exchange with other MGs. Let us denote this power exchange by $p_{j,p}(k) \in \mathbb{R}$. If $p_{j,p}(k) < 0$, then power is provided by MG_j, if $p_{j,p}(k) > 0$, then power is drawn by MG_j. This power is constrained by $\underline{p}_{j,p} \in \mathbb{R}_{<0}$ and $\overline{p}_{j,p} \in \mathbb{R}_{>0}$, i.e.,

$$\underline{p}_{j,p} \leq p_{j,p}(k) \leq \overline{p}_{j,p}. \tag{4a}$$

The local power balance of MG_j reads

$$p_{j,p}(k) + \sum_{i=1}^{g_j} p_{j,i}(k) + \sum_{i=g_j+1}^{g_j+l_j} w_{j,i}(k) = 0. \tag{4b}$$

2.2 *Power exchange over the electric grid*

The transmission network is modelled as an undirected graph $\mathcal{G} = (N, E)$, where each node in N represents an MG, i.e., $|N| = n$. Each element in E represents a power line that connects two nodes, i.e., two MGs. Let us denote the power exchange between two connected nodes oriented from $j \in N$ to $m \in N$ as $p_{jm}(k)$.

The power provided or consumed by MG_j via the PCC is linked to the power exchange with its neighbours by

$$p_{j,p}(k) = \sum_{m \in N_j} p_{jm}(k), \quad \forall j \in N. \tag{5}$$

We assume that the transmission network is composed of power lines with reactance to resistance ratio much greater than one. Therefore, line losses are assumed to be negligible (see, e.g., [1, 13, 15]). For lossless lines, we then have that $p_{jm}(k) = -p_{mj}(k)$ and that a global power balance of the form $\sum_{j \in N} p_{j,p}(k) = 0$ holds.

3 Conditional cooperation (CC) MPC for interconnected MGs

In MPC, control actions are derived by solving a finite horizon optimization problem [16]. From the solution, typically, the first value of the control actions is applied to the plant. At the next execution, initial conditions and forecasts are updated and the procedure is repeated.

3.1 Control objectives

The control objective of MG_j is given by

$$\ell_j(p_j,\delta_j)=\sum_{i\in C_j}\ell_{j,c}(p_{j,i},\delta_{j,i})+\sum_{i\in R_j}\ell_{j,r}(p_{j,i}) + \sum_{i\in S_j}\ell_{j,s}(p_{j,i})+\ell_{j,p}(p_{j,p}), \quad (6)$$

with $p_j=[\mathrm{vec}(p_{j,i})^\top_{i\in\mathbb{N}_{g_j}}\ p_{j,p}]^\top$, $\delta_j=\mathrm{vec}(\delta_{j,i})_{i\in C_j\cup S_j}$. The costs of the conventional, storage and renewable units are $\ell_{j,c}(\cdot,\cdot)\in\mathbb{R}_{\geq 0}$, $\ell_{j,s}(\cdot)\in\mathbb{R}_{\geq 0}$ and $\ell_{j,r}(\cdot)\in\mathbb{R}_{\geq 0}$. The cost associated with power exchange is $\ell_{j,p}(\cdot)\in\mathbb{R}$.

The objective function of conventional unit $i\in C_j$ is based on running costs. These can be approximated by

$$\ell_{j,c}(p_{j,i},\delta_{j,i})=a_{j,i}\delta_{j,i}+a'_{j,i}p_{j,i}+a''_{j,i}p_{j,i}^2, \quad (7a)$$

with weights $a_{j,i}\in\mathbb{R}_{>0}$, $a'_{j,i}\in\mathbb{R}_{>0}$ and $a''_{j,i}\in\mathbb{R}_{>0}$ [27]. Note that the square term in (7a) allows to model a maximum cost efficiency at power values below $\overline{p}_{j,i}$.

It is desirable to maximise infeed of RES. Therefore, $\ell_{j,r}(\cdot)$ penalises a limitation of available infeed $w_{j,i}$. With weight $a_{j,i}\in\mathbb{R}_{>0}$, it can be expressed for all $i\in R_j$ by

$$\ell_{j,r}(p_{j,i})=a_{j,i}(p_{j,i}-w_{j,i})^2. \quad (7b)$$

Charging or discharging at high power can have a negative impact on the ageing of batteries [21]. This is modelled for storage unit $i\in S_j$ with weight $a_{j,i}\in\mathbb{R}_{>0}$ by

$$\ell_{j,s}(p_{j,i})=a_{j,i}p_{j,i}^2. \quad (7c)$$

With selling/buying price $a_{j,p}\in\mathbb{R}_{>0}$ and trading cost weight $a'_{j,p}\in\mathbb{R}_{>0}$, the power exchange cost reads

$$\ell_{j,p}(p_{j,p})=a_{j,p}p_{j,p}+a'_{j,p}|p_{j,p}|. \quad (7d)$$

3.2 Central CC MPC

We assume that all MGs are capable of running in islanded mode. Therefore, motivated by [25], a condition is included which ensures that each MGs' connected operation cost does not exceed its islanded operation cost. Before posing different MPC problems, we need to introduce the sets of feasible power and energy values.

Let us collect the forecasts of available renewable power and load in $\hat{w}_j(k+h|k)=\mathrm{vec}(\hat{w}_{j,i}(k+h|k))_{i\in R_j\cup L_j}$. With this, the set of feasible power values of MG_j can be formulated as

$$\mathcal{P}_j(\hat{w}_j,\delta_j)=\{p_j\in\mathbb{R}^{g_j+1}|\text{Eqs. (1), (2a), (3), (4) hold and } w_j=\hat{w}_j\}.$$

Let us further define the vectors $x_j(k)=\mathrm{vec}(x_{j,i}(k))_{i\in S_j}$, $p_{j,s}(k)=\mathrm{vec}(p_{j,i}(k))_{i\in S_j}$, $\delta_{j,s}(k)=\mathrm{vec}(\delta_{j,i}(k))_{i\in S_j}$ and $b_j(\delta_{j,s}(k))=\mathrm{vec}(\eta_{j,i}\delta_{j,i}(k)+\frac{1}{\eta_{j,i}}(1-\delta_{j,i}(k)))_{i\in S_j}$. For MG_j, energy constraint (2c), can be captured by the set

$$\mathcal{X}_j=\{x_j\in\mathbb{R}^{|S_j|}\mid \underline{x}_{j,i}\leq x_{j,i}\leq\overline{x}_{j,i},\ \forall i\in S_j\}.$$

3.2.1 Condition for cooperation (Islanded MGs)

The optimal islanded operation cost for each MG is calculated by solving an MPC problem with zero PCC power. The decision variables are the power values of all units, the on/off condition of the conventional units and the charge/discharge mode of the storage units. At MG_j, let us collect the decision variables over prediction horizon $H\in\mathbb{N}$ in $\mathbf{p}_j=[p_j(k|k)\ \cdots\ p_j(k+H|k)]$, and $\boldsymbol{\delta}_j=[\delta_j(k|k)\ \cdots\ \delta_j(k+H|k)]$. Here, $p_j(k+h|k)$ refers to a prediction for time $k+h$, $h\in\mathbb{N}_{[0,H]}$, performed at time k.

Consider a measurement of the stored energy x_j^0 and a forecast of the uncertain input $\hat{w}_j$ over prediction horizon H. Then, the MPC problem associated with the islanded operation of MG_j can be formulated as follows.

Problem 1 (Islanded MPC)

$$\min_{\mathbf{p}_j,\boldsymbol{\delta}_j} V_j(\mathbf{p}_j,\boldsymbol{\delta}_j)$$

with $V_j(\mathbf{p}_j,\boldsymbol{\delta}_j)=\sum_{h=0}^{H}\gamma^h\ell_j(p_j(k+h|k),\delta_j(k+h|k))$

subject to

$$x_j(k|k)=x_j^0, \quad (8a)$$
$$x_j(k+h+1|k)=x_j(k+h|k)-\mathrm{diag}(b_j(\delta_{j,s}(k+h|k)))T_s p_{j,s}(k+h|k), \quad (8b)$$
$$x_j(k+h+1|k)\in\mathcal{X}_j, \quad (8c)$$
$$p_j(k+h|k)\in\mathcal{P}_j(\hat{w}_j(k+h|k),\delta_j(k+h|k)), \quad (8d)$$
$$p_{j,p}(k+h|k)=0, \quad (8e)$$

for all $h\in\mathbb{N}_{[0,H]}$ *where* $\gamma\in(0,1]$ *is a discount factor.*

Let us denote the minimum of Problem 1 as V_j^I. It is desired that no MG has a disadvantage (in the form of higher costs) from grid-connected over islanded operation. For each MG_j, this can be encoded in the set

$$\mathcal{C}_j(\boldsymbol{\delta}_j, V_j^I) = \{\mathbf{p}_j \in \mathbb{R}^{(g_j+1)\times(H+1)} \mid V_j(\mathbf{p}_j, \boldsymbol{\delta}_j) \leq V_j^I\}.$$

3.2.2 Conditional cooperation MPC

The MPC problem decides the power of each DU in each MG along with the power exchange between the MGs. Let us define $\mathbf{P} = \mathrm{vec}(\mathbf{p}_j)_{j\in N}$ and $\boldsymbol{\delta} = \mathrm{vec}(\boldsymbol{\delta}_j)_{j\in N}$. The MPC problem for a network of interconnected MGs can then be formulated as follows.

Problem 2 (Central CC MPC)

$$\min_{\mathbf{P},\boldsymbol{\delta}} V(\mathbf{P}, \boldsymbol{\delta})$$

$$\textit{with} \quad V(\mathbf{P}, \boldsymbol{\delta}) = \sum_{j\in N} V_j(\mathbf{p}_j, \boldsymbol{\delta}_j)$$

subject to (8a)–(8d) *and*

$$0 = \sum_{j\in N} p_{j,p}(k+h|k), \tag{9a}$$

$$\mathbf{p}_j \in \mathcal{C}_j(\boldsymbol{\delta}_j, V_j^I), \tag{9b}$$

for all $h \in \mathbb{N}_{[0,H]}$, $j \in N$.

In Problem 2, (9b) ensures that the cost of each MG_j in interconnected operation does not exceed the cost of islanded operation. Problem 2 can be cast as a MIQCP, which is known to be NP-hard. Its computational complexity increases significantly with the number of MGs. One approach to solve Problem 2 in a distributed manner is by relaxing binary variables to continuous ones [6]. This, however, does not automatically yield a feasible solution to the original problem (mostly due to (3)). As an alternative, we propose an algorithm that ensures feasibility and can be implemented in a distributed manner.

4 Feasible decomposition for solving CC MPC

The main idea of the proposed feasible-decomposition-based (FD-based) algorithm is to iteratively solve a sequence of feasible intermediate problems such that the cost in the original problem is non-increasing. As stated in Algorithm 1, in our scheme, first a subproblem with fixed binary variables is solved to update the PCC power. Then, another subproblem with fixed PCC power is solved to update the binary variables. This scheme is repeated until a termination criterion is met.

4.1 Subproblem with fixed binary variables

At iteration q, the fixed binaries of MG_j are denoted $\boldsymbol{\delta}_j^q$. With $\boldsymbol{\delta}^q = \mathrm{vec}(\boldsymbol{\delta}_j^q)_{j\in N}$, we modify Problem 2 as follows.

Algorithm 1 Feasible decomposition for CC MPC

Initialisation: Find feasible values of $\mathbf{P}^1$ and $\boldsymbol{\delta}^1$, e.g., by solving Problem 1.
for $q = 1, \ldots, q^{\max}$
(1) Solve Problem 3 with fixed binary variables $\boldsymbol{\delta} = \boldsymbol{\delta}^q$ to obtain the PCC power $\tilde{p}_{j,p}^q(k+h|k)$ for all $j \in N$.
(2) Check termination criterion. If not met, proceed with (3).
(3) Solve Problem 4 for all MG_j with fixed PCC power $p_{j,p}(k+h|k) = \tilde{p}_{j,p}^q(k+h|k)$ to obtain $\mathbf{P}^{q+1}, \boldsymbol{\delta}^{q+1}$.
end: Output $\mathbf{P}^q$, $\boldsymbol{\delta}^q$.

Problem 3 (Modified CC MPC)

$$\min_{\mathbf{P}} \sum_{j\in N} V_j(\mathbf{p}_j, \boldsymbol{\delta}_j)$$

subject to (8a)–(8d), (9) *for all* $h \in \mathbb{N}_{[0,H]}$, $j \in N$ *with*

$$\boldsymbol{\delta} = \boldsymbol{\delta}^q. \tag{10}$$

Because of (10), constraint (9b) becomes convex and Problem 3 can be cast as a convex second order cone problem which can be solved efficiently using interior-point methods. At iteration $q = 1$, a feasible choice of $\boldsymbol{\delta}^1$ can be obtained from Problem 1. Note that Problem 3 can be solved distributedly as illustrated in Section 5.

4.2 Subproblem with fixed PCC power values

In Problem 2, (9a) accounts for the coupling between the MGs using only the PCC power of the different MGs. Fixing the PCC power at iteration q to the optimal result from Problem 3, which is part of $\widetilde{\mathbf{P}}^q$, enables a decomposition into $n = |N|$ subproblems. In Algorithm 1, we use the solutions to these subproblems, which can be stated as follows, to update the binary variables.

Problem 4 (Binary update CC MPC)

$$\min_{\mathbf{p}_j,\boldsymbol{\delta}_j} V_j(\mathbf{p}_j, \boldsymbol{\delta}_j)$$

subject to (8a)–(8d) *and*

$$p_{j,p}(k+h|k) = \tilde{p}_{j,p}^q(k+h|k), \tag{11}$$

for all $h \in \mathbb{N}_{[0,H]}$.

Remark 2 (Computational complexity) *Problem 4 can be solved for each MG separately. This is beneficial in two ways. (i) Problem 4 allows for a decentralized solution on different computing nodes. (ii) To solve Problem 4 for all MGs,* $\sum_{j=1}^n 2^{(H+1)(|C_j|+|S_j|)}$ *possible combinations of binary variables need to be considered. In contrast, for Problem 2 this number is* $2^{\sum_{j=1}^n (H+1)(|C_j|+|S_j|)}$*, which is typically much higher.*

Proposition 1 *Given decision variables $\mathbf{P}^q$ and $\boldsymbol{\delta}^q$, obtained by solving Problem 4 and given the intermediate result $\widetilde{\mathbf{P}}^q$ obtained by solving Problem 3 for $\boldsymbol{\delta}^q$. Also given $\mathbf{P}^{q+1}$ and $\boldsymbol{\delta}^{q+1}$, obtained by solving Problem 4 using $\widetilde{\mathbf{P}}^q$, which includes $\tilde{p}^q_{j,p}(k+h|k)$ for all $h \in \mathbb{N}_{[0,H]}$ and all $j \in N$. Then, we have that $V(\mathbf{P}^{q+1}, \boldsymbol{\delta}^{q+1}) \leq V(\mathbf{P}^q, \boldsymbol{\delta}^q)$.*

Proof: $\mathbf{P}^q$ is a feasible solution to Problem 3. Therefore, $V(\widetilde{\mathbf{P}}^q, \boldsymbol{\delta}^q) \leq V(\mathbf{P}^q, \boldsymbol{\delta}^q)$. Moreover, $(\widetilde{\mathbf{P}}^q_j, \boldsymbol{\delta}^q_j)$ is a feasible solution to Problem 4. Therefore, $V_j(\mathbf{P}^{q+1}_j, \boldsymbol{\delta}^{q+1}_j) \leq V_j(\widetilde{\mathbf{P}}^q_j, \boldsymbol{\delta}^q_j)$ and hence $V(\mathbf{P}^{q+1}, \boldsymbol{\delta}^{q+1}) \leq V(\mathbf{P}^q, \boldsymbol{\delta}^q)$. □

Problem 4, i.e., step (3) of Algorithm 1, can have multiple optimal solutions which exhibit different binary variables. At MG_j, let us denote the set of optimal binary variables by B^q_j. Then, the combined set of binary variables of all MGs is $B^q = B^q_1 \times B^q_2 \cdots \times B^q_n$.

Proposition 2 (Termination criterion) *Algorithm 1 can terminate if $V(\mathbf{P}^q, \boldsymbol{\delta}^q) = V(\widetilde{\mathbf{P}}^q, \boldsymbol{\delta}^q)$ holds $\forall \boldsymbol{\delta}^q \in B^q$. Moreover, it terminates in a finite number of iterations.*

Proof: For fixed $\boldsymbol{\delta}^q$, the cost $V(\cdot,\cdot)$ is strongly convex. Therefore, $V(\mathbf{P}^q, \boldsymbol{\delta}^q) = V(\widetilde{\mathbf{P}}^q, \boldsymbol{\delta}^q) \iff \mathbf{P}^q = \widetilde{\mathbf{P}}^q$. Consequently, for $\widetilde{\mathbf{P}}^q$ (which is identical to the previous result of Problem 4, $\mathbf{P}^q$) Problem 4 yields the same set B^q. Hence, the result of Algorithm 1 does not change in subsequent iterations and it can therefore be terminated.

Because of the termination criterion, no repeated combinations of binary decision variables that yield the same cost will occur. Moreover, the number of combinations of binary decision variables is finite. Consequently, the number of iterations of Algorithm 1 is finite. □

Corollary 1 *If the binary variables at iteration q are part of an optimal solution to Problem 2, i.e., $\boldsymbol{\delta}^q = \boldsymbol{\delta}^\star$, then the power obtained by solving Problem 3 at the next iteration is the optimal power $\mathbf{P}^\star$ because the cost $V(\cdot,\cdot)$ is strongly convex for fixed $\boldsymbol{\delta}^q$. Similarly, if the PCC power $\tilde{p}^q_{j,p}(k+h|k)$ is part of an optimal solution to Problem 2, then Problem 4 provides an optimal $\boldsymbol{\delta}^{q+1} = \boldsymbol{\delta}^\star$.*

Remark 3 *It can be cumbersome to find all elements of B^q. This motivates a greedy search that assumes only one element in B^q. The termination criterion (see Proposition 2) then becomes a check for equality of $\mathbf{P}^q$ and $\widetilde{\mathbf{P}}^q$.*

5 Distributed solution for feasible-decomposition-based algorithm

In Algorithm 1, the binary variables are updated in a decentralised manner using Problem 4. To obtain a distributed implementation of the overall algorithm, we also need to distributedly solve Problem 3. In this section, we provide an example of a distributed solution to Problem 3 which is based on [4] and only requires communication between neighbouring MGs. Note that the algorithm serves as an example and could be easily replaced by other distributed optimization approaches, e.g., the alternating direction method of multipliers used in [3].

In an MG, the PCC power is identical to the sum of power exchanged with all neighbouring MGs (see (5)). Therefore, we can equivalently state Problem 3 using the power exchange $p_{j,t}(k) = \text{vec}(p_{jm}(k))_{m \in N_j}$ and the fact that $0 = p_{jm}(k) + p_{mj}(k)$ for all $\{j,m\} \in E$ (see Section 2.2) instead of the PCC power $p_{j,p}$. Let us define $\mathbf{p}_{j,t} = [p_{j,t}(k|k) \ \cdots \ p_{j,t}(k+H|k)]$ and $\widetilde{\mathbf{p}}_{j,t} = [\tilde{p}_{j,t}(k|k) \ \cdots \ \tilde{p}_{j,t}(k+H|k)]$ for all $j \in N$ with $\tilde{p}_{j,t}(k) = \text{vec}(p_{mj}(k))_{m \in N_j}$ as well as $\mathbf{p}_t = [\mathbf{p}_{1,t}^\top \ \cdots \ \mathbf{p}_{n,t}^\top]$ such that we can rewrite Problem 3 as follows.

Problem 5 (Modified CC MPC with power flow)

$$\min_{\mathbf{P},\mathbf{p}_t} \sum_{j \in N} V_j(\mathbf{p}_j, \boldsymbol{\delta}_j)$$

subject to (8a)–(8d), (9b) *and*

$$p_{j,p}(k+h|k) = \sum_{m \in N_j} p_{jm}(k+h|k), \quad \forall j \in N, \tag{12a}$$

$$0 = \mathbf{p}_{j,t} + \widetilde{\mathbf{p}}_{j,t}, \tag{12b}$$

for all $h \in \mathbb{N}_{[0,H]}$, $j \in N$ with $\boldsymbol{\delta}_j = \boldsymbol{\delta}^q_j$.

One advantage of Problem 5 over Problem 3 is that it does not requite constraint (9a). This enables the use of distributed algorithms that rely on peer-to-peer communication and work without a central coordinator.

Let us define $\mathbf{p}_{jm} = [p_{jm}(k|k) \ \cdots \ p_{jm}(k+H|k)]$ and the Lagrangian variable $\lambda_{jm}(k) \in \mathbb{R}$ for all $\{j,m\} \in E$. Let us futher define $\boldsymbol{\lambda}_j = [\lambda_j(k|k) \ \cdots \ \lambda_j(k+H|k)]$ with entries $\lambda_j(k) = \text{vec}(\lambda_{jm}(k))_{m \in N_j}$ for all $j \in N$ as well as $\boldsymbol{\lambda} = [\boldsymbol{\lambda}_1^\top \ \cdots \ \boldsymbol{\lambda}_n^\top]$. With (12b) and fixed parameter $\rho > 0$, the augmented Lagrangian (AL) of Problem 5 is

$$L_\rho(\mathbf{P}, \mathbf{p}_t, \boldsymbol{\lambda}) = \sum_{j \in N} \big(f_j(\mathbf{p}_j, \mathbf{p}_{j,t}) + \langle \boldsymbol{\lambda}_j, \mathbf{p}_{j,t} + \widetilde{\mathbf{p}}_{j,t} \rangle + \rho/2 \|\mathbf{p}_{j,t} + \widetilde{\mathbf{p}}_{j,t}\|^2 \big). \tag{13}$$

Here, $f_j(\mathbf{p}_j, \mathbf{p}_{j,t})$ includes the objective $V_j(\mathbf{p}_j, \boldsymbol{\delta}_j)$ and the constraints (8a)–(8d), (9b) as well as $\boldsymbol{\delta}_j = \boldsymbol{\delta}^q_j$.

The dual problem associated with the AL is always differentiable, but does not exhibit a decomposable structure because of the last term in (13). Therefore, we adopt the distributed AL proposed in [4] which replaces this part by n separable quadratic terms. This results in a distributed AL algorithm that converges asymptotically [4].

Let us define the vectors $\hat{p}_{j,t}(k) = \text{vec}(\hat{p}_{mj}(k))_{m\in N_j}$ and $\hat{\mathbf{p}}_{j,t} = [\hat{p}_{j,t}(k|k) \ \cdots \ \hat{p}_{j,t}(k+H|k)]$ for each MG_j. We can then state the *local* augmented Lagrangian as

$$\hat{L}_{j,\rho}(\mathbf{p}_j, \mathbf{p}_{j,t}, \boldsymbol{\lambda}_j, \hat{\mathbf{p}}_{j,t}) = f_j(\mathbf{p}_j, \mathbf{p}_{j,t}) + \langle \boldsymbol{\lambda}_j, \mathbf{p}_{j,t} + \hat{\mathbf{p}}_{j,t}\rangle + \rho/2\|\mathbf{p}_{j,t} + \hat{\mathbf{p}}_{j,t}\|^2. \quad (14)$$

The separable approximate for the AL (13) is given by

$$\hat{L}_\rho(\mathbf{P}, \mathbf{p}_t, \boldsymbol{\lambda}) = \sum_{j\in N} \hat{L}_{j,\rho}(\mathbf{p}_j, \mathbf{p}_{j,t}, \boldsymbol{\lambda}_j, \hat{\mathbf{p}}_{j,t}). \quad (15)$$

At iteration $\nu \in \mathbb{N}$, $\hat{\mathbf{p}}^\nu_{j,t}$ is updated using the power exchange calculated at all neighbouring MG_m, $m \in N_j$, i.e., $\hat{\mathbf{p}}^\nu_{mj} = \mathbf{p}^\nu_{mj}$. Then, power and power exchange are updated by solving the local AL, i.e.,

$$(\mathbf{p}^{\nu+1}_j, \bar{\mathbf{p}}^\nu_{j,t}) \in \underset{\mathbf{p}_j,\mathbf{p}_{j,t}}{\text{argmin}}\, \hat{L}_{j,\rho}(\mathbf{p}_j, \mathbf{p}_{j,t}, \boldsymbol{\lambda}^\nu_j, \hat{\mathbf{p}}^\nu_{j,t}), \quad (16a)$$

$$\mathbf{p}^{\nu+1}_{j,t} = \mathbf{p}^\nu_{j,t} + \tau(\bar{\mathbf{p}}^\nu_{j,t} - \mathbf{p}^\nu_{j,t}), \quad (16b)$$

with $\tau \in \mathbb{R}_{>0}$. Then, MG_j communicates $\mathbf{p}^{\nu+1}_{jm}$ to all neighbouring MGs and updates the dual variables $\boldsymbol{\lambda}_j$. This is summarised in Algorithm 2. Note that Algorithm 2 terminates after a predefined number of iterations $\nu^{\max}$ or if the residual $\epsilon^\nu_j = \max(\|\bar{\mathbf{p}}^\nu_{j,t} - \mathbf{p}^\nu_{j,t}\|, \|\mathbf{p}^\nu_{j,t} - \hat{\mathbf{p}}^\nu_{j,t}\|)$ is smaller than a given bound ϵ_{term} at all nodes $j \in N$.

Algorithm 2 Accelerated distributed AL

Initialisation at time $k \in \mathbb{N}$:
- Select $\tau \in (0, \frac{1}{2})$ and $\rho > 0$.
- Set $\boldsymbol{\lambda}^0_j$ and $\hat{\mathbf{p}}^0_{j,t}$ to initial value, e.g., zero, for all $j \in N$.

for $\nu = 1, \ldots, \nu^{\max}$
  for all MG_j, $j \in N$ (in parallel)
  (1) Fix $\boldsymbol{\lambda}^\nu_j$, $\hat{\mathbf{p}}^\nu_{j,t}$ and solve

$$(\mathbf{p}^{\nu+1}_j, \bar{\mathbf{p}}^\nu_{j,t}) \in \underset{\mathbf{p}_j,\mathbf{p}_{j,t}}{\text{argmin}}\, \hat{L}_{j,\rho}(\mathbf{p}_j, \mathbf{p}_{j,t}, \boldsymbol{\lambda}^\nu_j, \hat{\mathbf{p}}^\nu_{j,t}).$$

  (2) Update power exchange variables using

$$\mathbf{p}^{\nu+1}_{j,t} = \mathbf{p}^\nu_{j,t} + \tau(\bar{\mathbf{p}}^\nu_{j,t} - \mathbf{p}^\nu_{j,t}).$$

  (3) Communicate $\mathbf{p}^{\nu+1}_{jm}$ to all neighbours $m \in N_j$.
  (4) Receive $\hat{\mathbf{p}}^{\nu+1}_{j,t}$ from neighbours.
  (5) Update Lagrangian variables using

$$\boldsymbol{\lambda}^{\nu+1}_j = \boldsymbol{\lambda}^\nu_j + \rho\tau(\mathbf{p}^{\nu+1}_{j,t} - \hat{\mathbf{p}}^{\nu+1}_{j,t}).$$

  (6) Terminate if $\epsilon^\nu_j \le \epsilon_{\text{term}}$, $\forall j \in N$.

end: Output optimal $p^\nu_{j,p}(k|k), \ldots, p^\nu_{j,p}(k+H|k)$.

Table 1
Parameters of MG_j, $j \in \mathbb{N}_4$.

Parameter	Value
$\text{vec}(\underline{p}_{j,c})_{c\in C_j, j\in\mathbb{N}_4}$	$[0.1 \quad 0.25 \quad 0.1 \quad 0.25]$ pu
$\text{vec}(\overline{p}_{j,c})_{c\in C_j, j\in\mathbb{N}_4}$	$[0.8 \quad 1 \quad 0.8 \quad 1]$ pu
$[\underline{p}_{j,p} \quad \overline{p}_{j,p}]$	$[-1 \quad 1]$ pu
$[\underline{p}_{j,s} \quad \overline{p}_{j,s}]$	$[-1 \quad 1]$ pu $\quad \forall s \in S_j$
$[\underline{p}_{j,r} \quad \overline{p}_{j,r}]$	$[0 \quad 2]$ pu $\quad \forall r \in R_j$
$[\underline{x}_{j,s} \quad \overline{x}_{j,s}]$	$[0 \quad 6]$ pu h $\quad \forall s \in S_j$
$\text{vec}(\eta_{j,s})_{j\in\mathbb{N}_4}$	$[0.95 \quad 0.9 \quad 0.95 \quad 0.9]$
$\text{vec}(a_{j,c})_{c\in C_j, j\in\mathbb{N}_4}$	$[0.44 \quad 0.33 \quad 0.33 \quad 0.47]$
$\text{vec}(a'_{j,c})_{c\in C_j, j\in\mathbb{N}_4}$	$[0.58 \quad 0.83 \quad 0.87 \quad 0.67]$ $1/\text{pu}$
$\text{vec}(a''_{j,c})_{c\in C_j, j\in\mathbb{N}_4}$	$[0.64 \quad 1 \quad 0.90 \quad 0.76]$ $1/\text{pu}^2$
$[a_{j,p} \quad a'_{j,p}]$	$[0.35 \quad 0.1]$ $1/\text{pu}$
$a_{j,s}$	0.1 $1/\text{pu}^2$ $\quad \forall s \in S_j$
$a_{j,r}$	1 $1/\text{pu}^2$ $\quad \forall r \in R_j$

6 Case study

In the case study, the topology in Figure 1 was considered. It is composed of four MGs that are connected by power lines. Each MG comprises a load, a renewable, a conventional and a storage unit. Parameters and weights of the objective are summarised in Table 1. Available renewable infeed was calculated from wind and irradiance data provided by [2]. For the load, a realistic pattern was generated based on real-world MG data.

The MPC provides power values to the units at a sampling rate of $T_\text{s} = 30$ min. Moreover, a prediction horizon of 6 h, i.e., $H = 12$ was chosen based on [6] and a naive persistence forecaster (see, e.g., [7, 11]) was used to predict load and available renewable infeed. The simulations were performed using MATLAB® 2015a. Optimization problems were formulated with YALMIP [12] and solved with Gurobi 7.5.2. The analysis was carried out for 336 simulation steps, i.e., one week.

6.1 Performance of FD-based algorithm

In this section, we assess convergence and open-loop costs of Algorithm 1. The input data to the controller was generated by performing closed-loop simulations with the MPC formulation given by Problem 2. This resulted in 336 samples of input data for which Algorithm 1 was executed using the greedy termination criterion discussed in Remark 3. As stated in Proposition 2, it converges to a sub-optimal feasible solution in a finite number of iterations. In Figure 2, it can be seen that the algorithm requires on average 3.3 iterations to converge and rarely (only 3.6 % of the cases) reaches the predefined maximum number of iterations $q^{\max} = 6$.

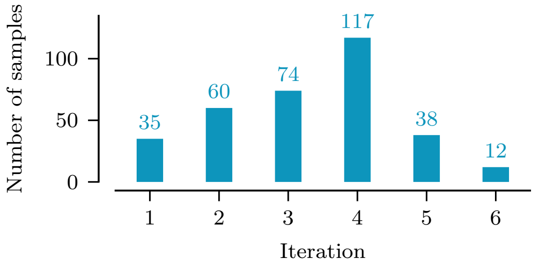

Fig. 2. Number of iterations required for termination of FD-based algorithm.

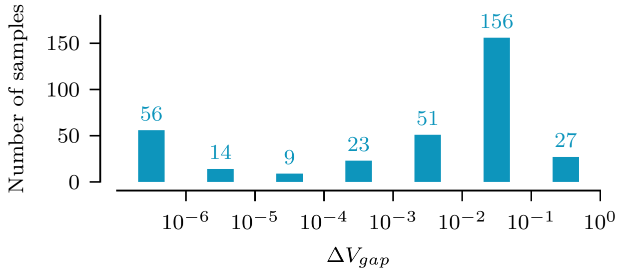

Fig. 4. Difference between cost obtained via Problem 2 and Algorithm 1.

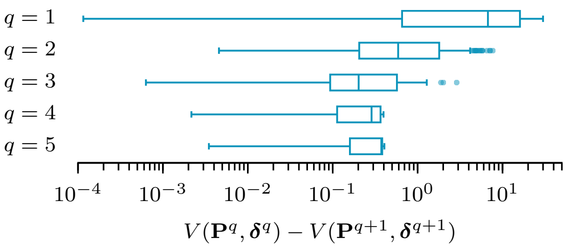

Fig. 3. Decrease of cost at each iteration of the FD-based algorithm.

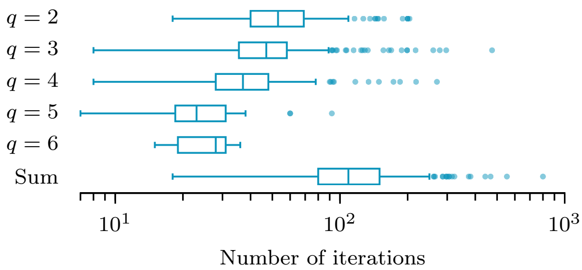

Fig. 5. Number of inner iterations of Algorithm 2 at each outer iteration of Algorithm 1 and overall sum at each execution of Algorithm 1.

Figure 3 shows the cost difference between two consecutive iterations. Here, the values with zero difference, i.e., those where the termination criterion came to bear, are omitted. It can be observed, that $V(\mathbf{P}^q, \boldsymbol{\delta}^q) - V(\mathbf{P}^{q+1}, \boldsymbol{\delta}^{q+1})$ is nonnegative due to the monotonic decrease of V (see Proposition 1).

In the context of this paper, we define the optimality gap as the difference between the optimal cost obtained by solving Problem 2 and the one resulting from the FD-based Algorithm 1. Corollary 1 discusses when this gap is zero. However, Corollary 1 does not include any information about the size of nonzero gaps. To assess these gaps, the measure

$$\Delta V_{\text{gap}} = {(V^{\text{Prob. 2}} - V^{\text{Alg. 1}})}/{\sum_{j \in \mathbb{N}_4} V_j^I}$$

is used to compare the cost obtained via Problem 2, $V^{\text{Prob. 2}}$, to the one obtained with Algorithm 1, $V^{\text{Alg. 1}}$. Figure 4 shows the empirical distribution of this gap for 336 samples of input data. It can be observed that for the majority of sample points, the gap is less than 0.1, i.e., the solution obtained via Algorithm 1 is relatively close to the one of the original Problem 2.

Now, we compare the time required to solve Problem 2 with Gurobi with the time required by Algorithm 1. A key benefit of the proposed algorithm is that the search space for binary decisions grows linearly with the number of MGs whereas it grows exponentially in Problem 2. This is reflected in the solve times of Gurobi which were on average 354 s for Problem 2, whereas the central version of Algorithm 1 always converged in less than 2 s.

6.2 Distributed FD-based algorithm

In this part, we analyse the distributed implementation of the FD-based algorithm using the same test data as in Section 6.1. The algorithm includes nested iterations, i.e., outer iterations of Algorithm 1 and inner iterations of Algorithm 2 which is used to solve Problem 3. We heuristically selected $\tau = 0.45$, $\rho = 100$ and $\epsilon_{\text{term}} = 10^{-3}$. Moreover, $\mathbf{p}_{j,t}^{\nu+1}$ and $\boldsymbol{\lambda}_j^{\nu+1}$ from the previous outer iteration are used as initial values for $\hat{\mathbf{p}}_{j,t}^0$ and $\boldsymbol{\lambda}_j^0$ to warm-start Algorithm 2.

Figure 5 illustrates the number of inner iterations required at outer iteration $q = 1, \ldots, 6$ as well as the sum over the entire execution of Algorithm 1. This sum is on average 126 with the maximum being 800. The solve time of each iteration of Algorithm 2 is less than 80 ms. When neglecting communication delays, the maximum compute time of the distributed FD-based Algorithm 1 (which includes Algorithm 2) and the binary updates in the form of Problem 4, is 75 s. This is much less than for the central approach which required on average 354 s to solve Problem 2 (see previous section). An even greater reduction of compute times can be expected with a larger number of interconnected MGs.

Most of the time, the average number of inner iterations decreases from one outer iteration to the next. Exceptions are probably caused by fixed values ρ and τ in presence of changing binary variables which modify Problem 5. Tuning ρ and τ to speed-up convergence will be part of future research.

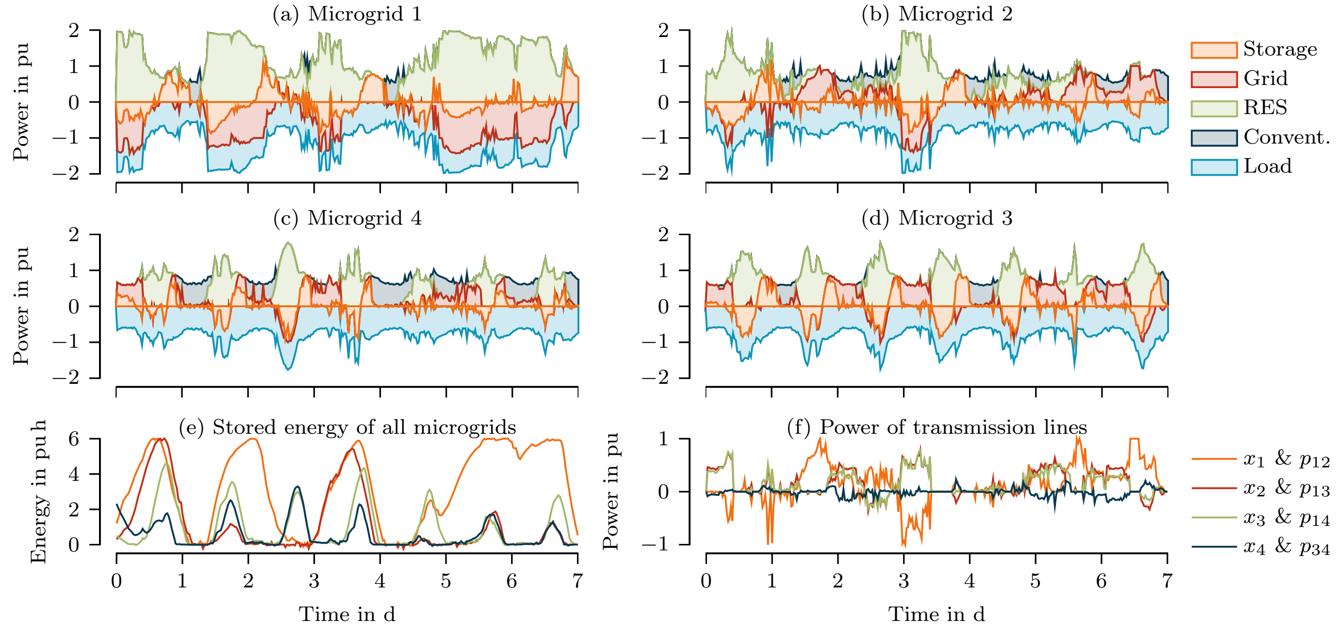

Fig. 6. Closed-loop simulation results obtained with distributed CC MPC approach.

6.3 *Closed-loop simulations*

In what follows, the closed-loop performance is studied for different controllers: (i) central CC MPC, i.e., Problem 2, (ii) distributed CC MPC, i.e., Algorithm 1, with Algorithm 2 in the inner loop, and (iii) islanded MPC, i.e., Problem 1.

Figure 6 visualizes the results of the simulation using the distributed CC MPC. MG_3 and MG_4 have photovoltaic based RES. Therefore, these MGs more often receive power from other MGs at night. Compared to islanded operation, the power exchange allows to significantly reduce the overall conventional generation. In Figure 6, it can be noted that the MGs are rarely selling power when their conventional generator is switched on. This is probably caused by a price for operating the conventional unit which is higher than the benefit from trading.

One objective when interconnecting MGs is to curtail renewable infeed as little as possible. To evaluate this desire, we introduce a performance indicator that represents the share of uncurtailed renewable infeed for all $j \in \mathbb{N}_4$, i.e.,

$$\mathrm{KPI}_j = 100\,\% \cdot \sum_{k=1}^{336} \sum_{i \in R_j} {p_{j,i}(k)}/{w_{j,i}(k)},$$

where $w_{j,i}(k)$ represents the available renewable infeed of MG_j. For each MG and the overall system KPI_j is shown in Table 2. It can be observed that KPI_j significantly increases when allowing for power exchange compared to islanded operation. Moreover, it can be noted that KPI_j of the central CC MPC and the distributed CC MPC are less than 1 % away from each other. Hence, the use of Algorithm 1 only comes with a minor reduction of renewable infeed compared to Problem 2.

The closed-loop cost determined via (6) is also summarised in Table 2. We can observe that the costs significantly decrease when power exchange is allowed. The centralised controller and the proposed distributed controller yield a cost decrease of 65.1 % and 63.1 %, respectively, compared to islanded operation. It can be further noted that the FD-based CC MPC results in a suboptimal operation compared to the central controller which leads to a slight increase in costs of 4 %. This, however, seems acceptable considering that the FD-based algorithm is distributed and computationally scalable.

Table 2
Percentage of uncurtailed renewable infeed, KPI_j, and closed-loop costs with different MPC approaches.

	KPI_j in %			Closed-loop costs		
	Centr.	FD-bas.	Islan.	Centr.	FD-bas.	Islan.
MG_1	86.9	86.2	44.4	25	19	445
MG_2	78.5	78.2	69.2	132	158	270
MG_3	86.6	86.7	82.3	110	99	249
MG_4	87.0	87.0	82.4	171	179	296
Sum	85.2	84.9	62.2	439	455	1260

7 Conclusions

In this paper, we proposed a conditional cooperation model predictive control strategy for the operation of interconnected MGs. The strategy preserves the self-interest of each MG by only trading power if this does not increase their individual costs. A feasible-decomposition-based algorithm was proposed to solve the associated mixed integer program. This algorithm decreases the cost monotonically and converges in a finite number of iterations. The algorithm was posed in a distributed way and solved by employing an augmented Lagrangian algorithm. In a comprehensive case study, properties of the algorithm, e.g., costs and compute times, were assessed. Moreover, it was demonstrated that the algorithm preserves the MGs' self-interests and decreases the operation costs.

Future work will focus on alternative strategies that reduce the number of binary combination in the presence of multiple optimal solutions to Problem 4. Furthermore, the inclusion of uncertainty as in [9, 10] and a dynamic update of the selling price as in [5] are planned to be investigated.

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